

CS180- Vectors

Note Title

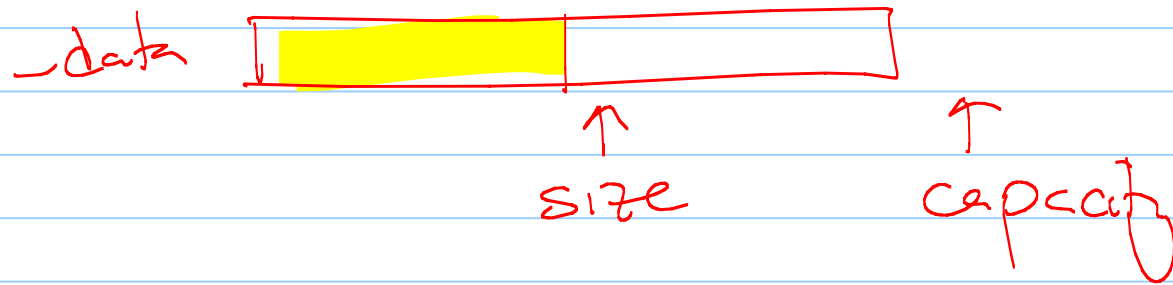
10/20/2011

Announcements

- Vector HW - due next Monday
- Lab tomorrow
- Review next Thursday, test next Friday

Vector :
"nice" version of array

implementation:



Runtime

insert: } $O(n)$
erase: }
~~push-back~~: }

operator [] : $O(1)$

push-back
amortized runtime of $O(1)$

Analysis

Consider push_back in a vector

Running time? (worst case)

$O(n)$

making k push_backs

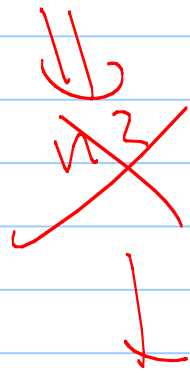
Amortization

Every time we have to rebuild the array we get a bunch of extra spots.

Need to formalize this idea:

amortization: finding average running time per operation over a long series of operation

n calls to $O(n)$ function



$O(n)$

$$\underbrace{O(1) \quad O(n) \quad O(1) \quad O(n)}_n = O(n^2)$$

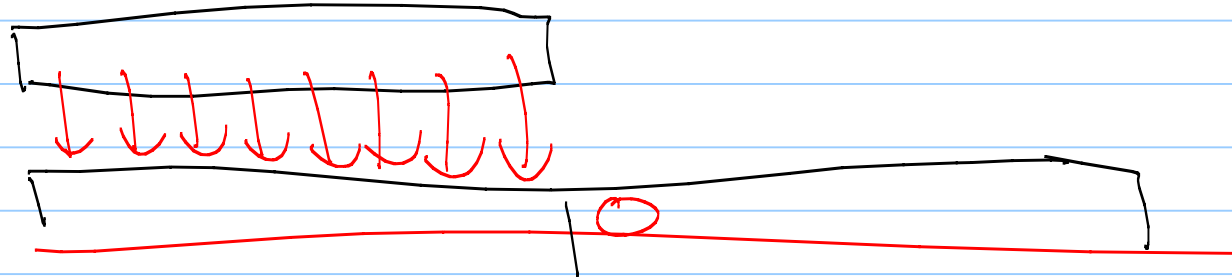
Claim: The total time to perform a series of n push-back operations into an initially empty vector is $O(n)$.

proof: Think of a bank account. Each constant time operation costs \$1 to run.

So each non-overflow push costs \$1.

Overflow inserts? $\$k$, where $k = \text{size}$

size k

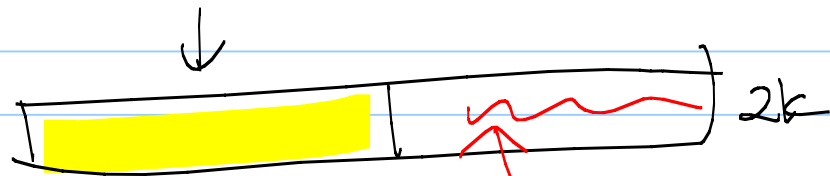
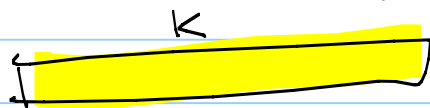


size $2k$

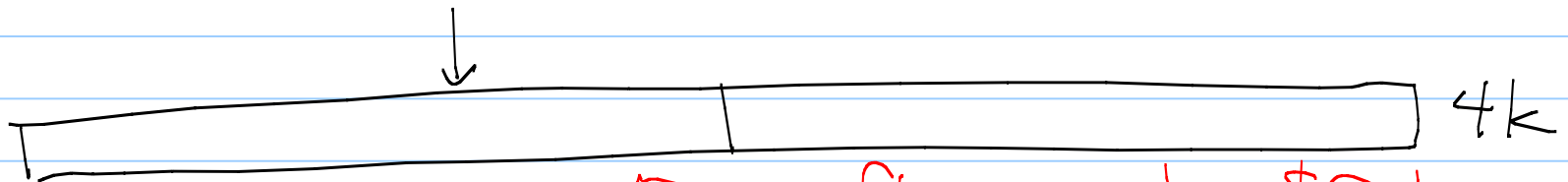
$\$k+1$

$10k?$

Key idea: overcharge the non-overflow pushes



k push_backs



overflow costs \$2k

$$3 \cdot k - 1 \cdot k = 2k$$



Analysis: array has 2^i elements in it
& needs to be doubled

Last double had 2^{i-1} so a
total of 2^{i-1} new things have
been inserted since then

Each gave \$3, & cost \$1.

Account: $3 \cdot 2^{i-1} - 2^{i-1} = 2 \cdot 2^{i-1} = 2^i$,
"paying" for overflow insert in
this round.