CS180 - Huffman trees

Note Title 11/16/2011

Announcements

- Midterm 2 in 1 week!
  - practice midterm in center table
  - review Friday (& no lab this week)
- AVL HW due Thursday
- Next HW will be posted, but not due until a week after the exam (in pairs)

Idea

We want to transmit information using as few bits as possible.

Standard ASCII: 8 bits

(Extended - 64 bits)

Encode: 40,000 characters × 8 bits

So- how can we do better?

What if we don't use every character?

Hello: 5x8 bits

More common characters should use fewer bits.

Problem:

Confusion while parsing:

Codes:

- E: 11
- A: 00
- S: 01
- T: 10
- R: 110
- M: 001
- B: 010
- N: 100

Decode this:

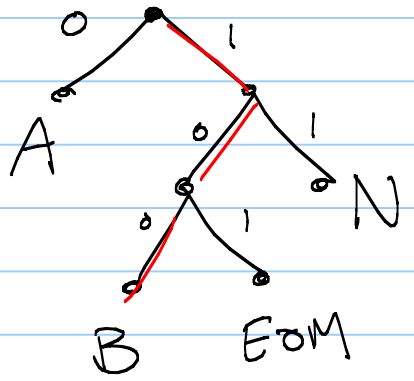
11001

EM

or

RS

Prefix-free codes



An unambiguous way to send information when we have characters that are not of a fixed length.

No letter's code is the prefix of another letter.

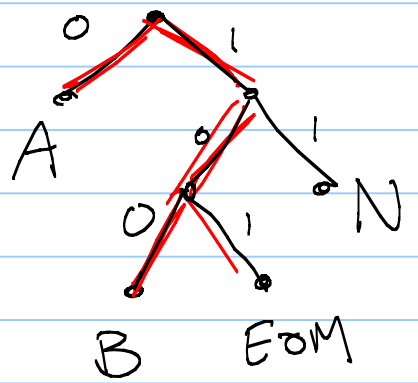
Encode: BAN

100911

Decode:

1000 110 1111 0101

B A N A N A (EOM)



13 bits

versus

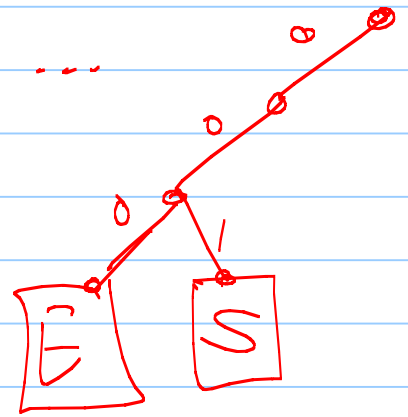
7 x 8 = 56 bits

So how do we do this? With exact frequency counts!

This sentence contains three a's, three c's, two d's, twenty-six e's, five f's, three g's, eight h's, thirteen i's, two l's, sixteen n's, nine o's, six r's, twenty-seven s's, twenty-two t's, two u's, five v's, eight w's, four x's, five y's, and only one z.

A	C	D	E	F	...
3	3	2	26	5	

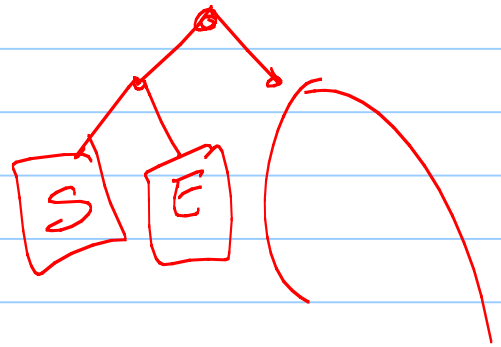
Goal: Make a tree:



Using frequency counts, build one of those trees.

A	C	D	E	F	G	H	I	L	N	O	R	S	T	U	V	W	X	Y	Z
3	3	2	26	5	3	8	13	2	16	9	6	27	22	2	5	8	4	5	1

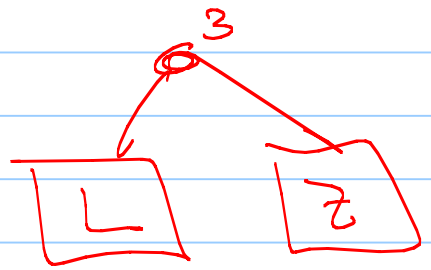
Which ones should get few bits?



Huffman's algorithm

Take the two least frequent characters.

Merge them into 1 letter, which becomes a new "leaf".

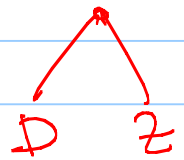


Example:

A	C	D	E	F	G	H	I	L	N	O	R	S	T	U	V	W	X	Y	Z
3	3	2	26	5	3	8	13	2	16	9	6	27	22	2	5	8	4	5	1

Merge D & Z:

A	C	E	F	G	H	I	L	N	O	R	S	T	U	V	W	X	Y	DZ
3	3	26	5	3	8	13	2	16	9	6	27	22	2	5	8	4	5	3

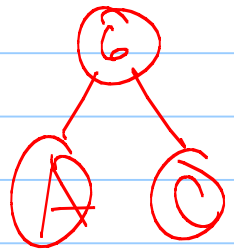


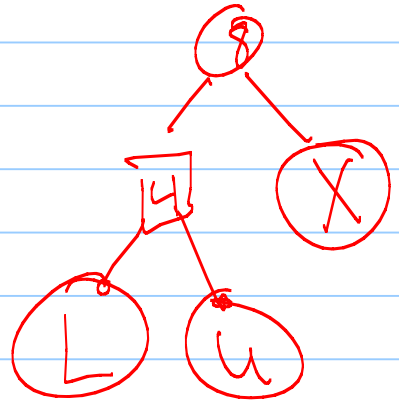
A	C	E	F	G	H	I	L	N	O	R	S	T	U	V	W	X	Y	Z
3	3	26	5	3	8	13	2	16	9	6	27	22	2	5	8	4	5	3

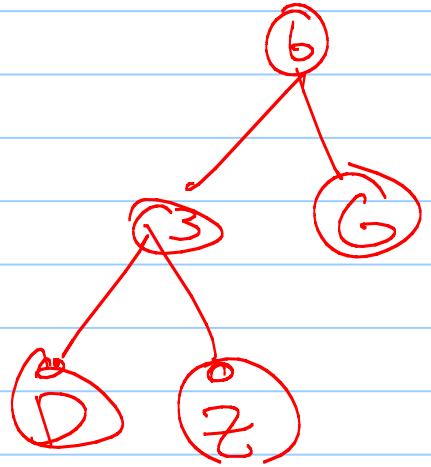
LN	AC	GDZ
4	6	6

Next?

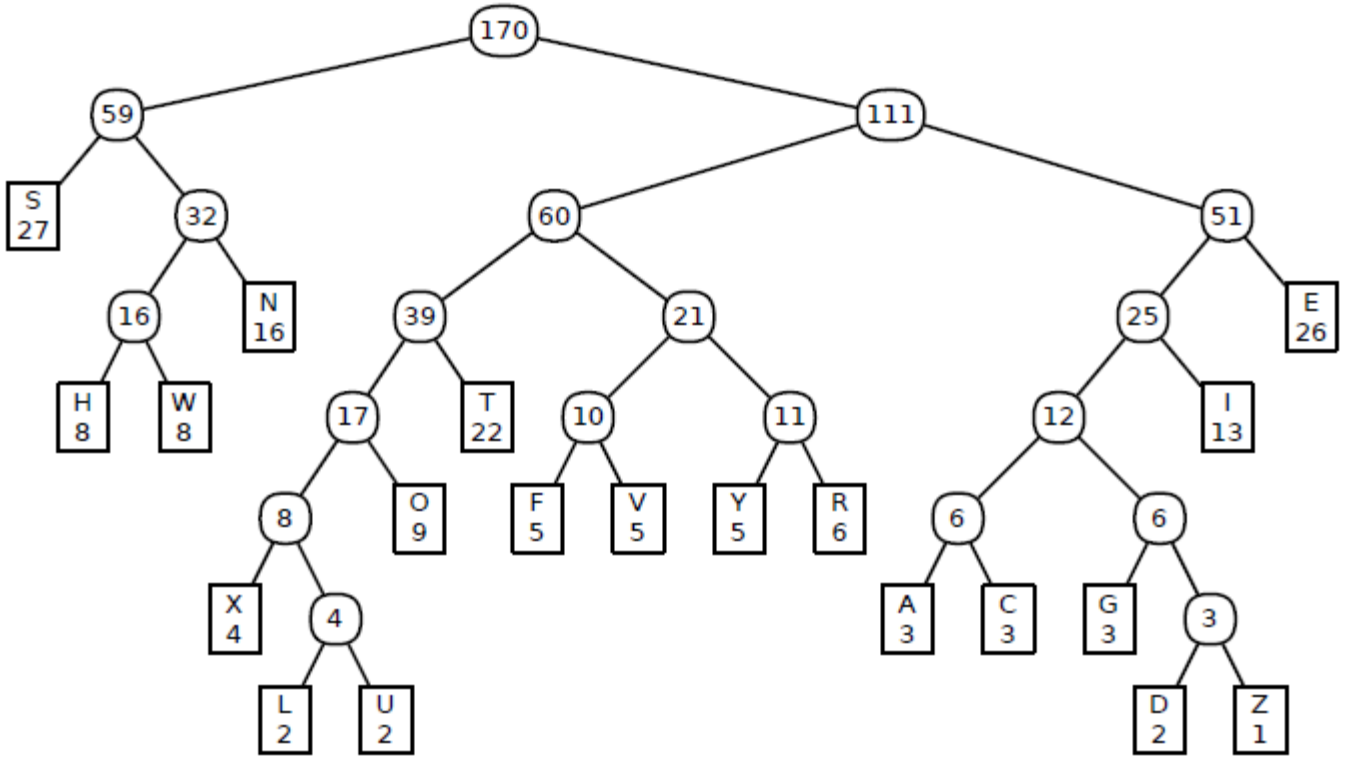
XLN
8



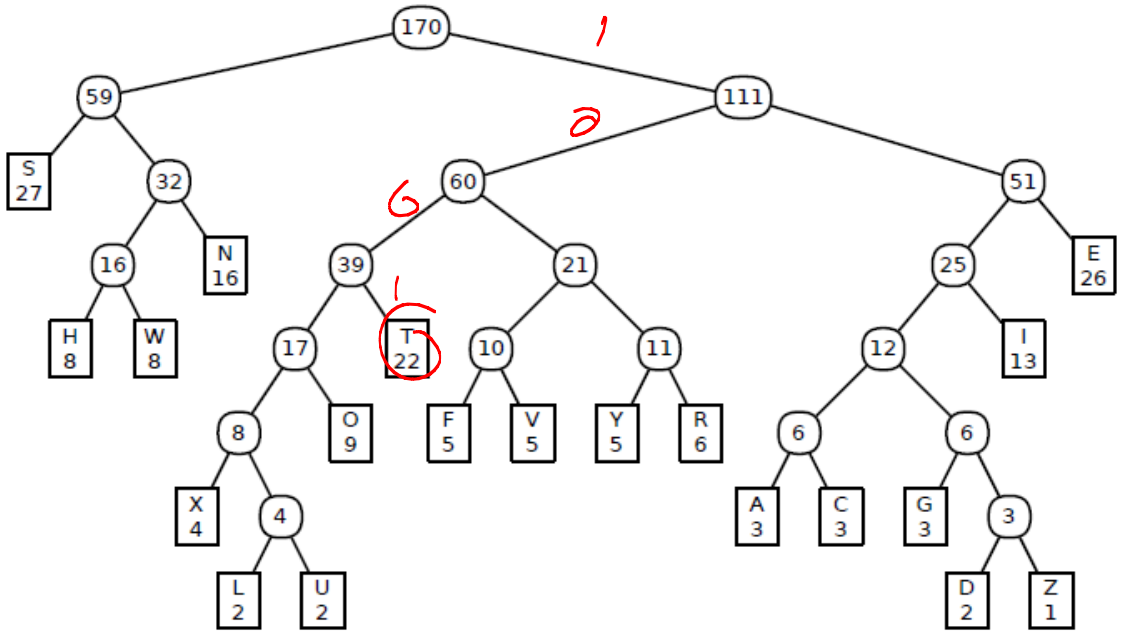




In end, build a tree:



Using the tree:



1001 0100 1101 00 00 111 011 1001 111 011 110001 111 110001 10001 011 1001 110000 1101 ...

T H I S S E N T E N C E C O N T A I

How many bits?

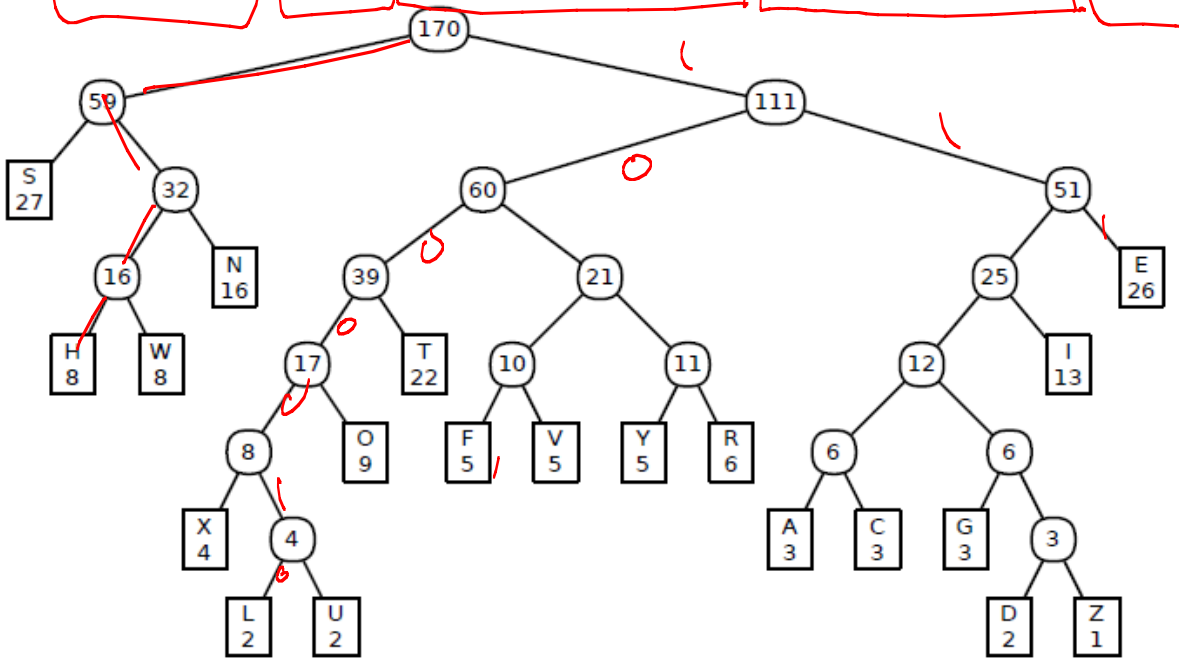
char.	A	C	D	E	F	G	H	I	L	N	O	R	S	T	U	V	W	X	Y	Z
freq.	3	3	2	26	5	3	8	13	2	16	9	6	27	22	2	5	8	4	5	1
depth	6	6	7	3	5	6	4	4	7	3	4	4	2	4	7	5	4	6	5	7
total	18	18	14	78	25	18	32	52	14	48	36	24	54	88	14	25	32	24	25	7

total = 646 bits

How many bits would ASCII use to send these 170 letters?

170 × 8 = 1360

Exercise: 0 1 0 0 1 1 1 1 0 0 0 0 1 0 1 0 0 0 0 1 0 1 0 0 0 1



Message? HELLO

How many bits? 26 versus 5x8=40

Thm: Huffman codes are optimal, in the sense that they use the fewest # of bits possible.

(Go take 314 to see the proof, or read supplemental notes on the schedule page.)

This is a greedy algorithm.

Next program: Decode

Given an input which describes a tree and a set of bits which are a message:

1) Create the tree (our BinaryTree.h)
2) Use it to decode the message