Math 135 - Set identities + proofs

Note Title 9/10/2012

Announcements

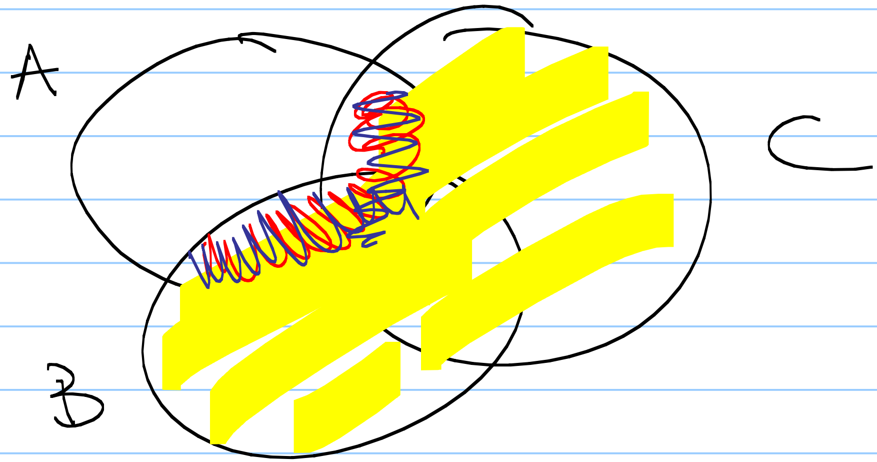
- HW due Friday
- Next will be posted Friday
- Office hours tomorrow are moved to 9-10am

Set identities (p. 130)

Thm: For all sets A, B, & C,

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

(so $\cap$ distributes over $\cup$)



Proof: Show ① $A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$

and $(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$

① $\forall x$, if $x \in A \cap (B \cup C)$, the $x \in (A \cap B) \cup (A \cap C)$

Spps $x \in A \cap (B \cup C)$

$\Rightarrow [x \in A] \wedge [x \in (B \cup C)]$

$\Rightarrow (x \in A) \wedge (x \in B \vee x \in C)$ } dfns

use ch 1.3 identity

$p \wedge (q \vee r)$

$\equiv (p \wedge q) \vee (p \wedge r)$

$\Rightarrow (x \in A \wedge x \in B) \vee (x \in A \wedge x \in C)$

$\Downarrow$

$\Rightarrow (x \in A \cap B) \vee (x \in A \cap C)$ } dfns

$= x \in (A \cap B) \cup (A \cap C)$

② $(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$

$\forall x$, if $x \in$ then $x \in$

take $x \in (A \cap B) \cup (A \cap C)$

(use dfn)

$\Rightarrow x \in A \cap B$ or $x \in A \cap C$

$\Rightarrow [x \in A$ and $x \in B]$ or $[x \in A$ and $x \in C]$

use same identity!

$\Rightarrow x \in A$ and $[x \in B$ or $x \in C]$

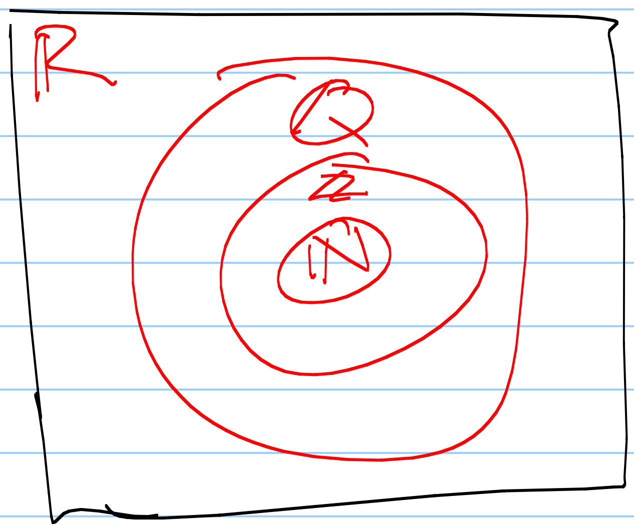
$\Rightarrow x \in A \cap (B \cup C)$

$\blacksquare$

The Universe

Most of the time, our sets will come from a single large set called the universe.

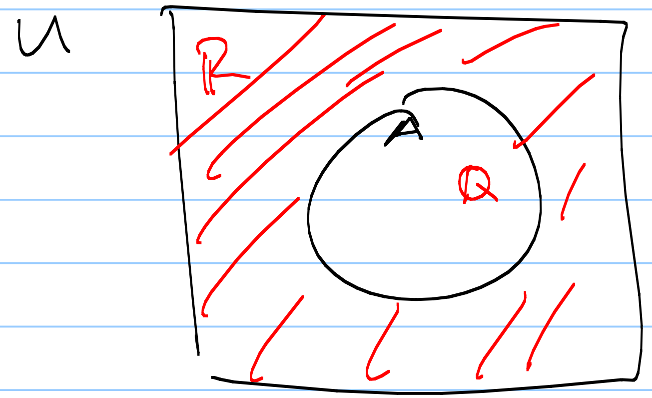
Ex:



Complement

Relative to U, the complement of A is

$$\bar{A} = U - A = \{x : x \notin A\}$$



Ex: $\mathbb{Z} - \mathbb{N}$

$\mathbb{R} - \mathbb{Q} = \overline{\mathbb{Q}}$

DeMorgan's Law

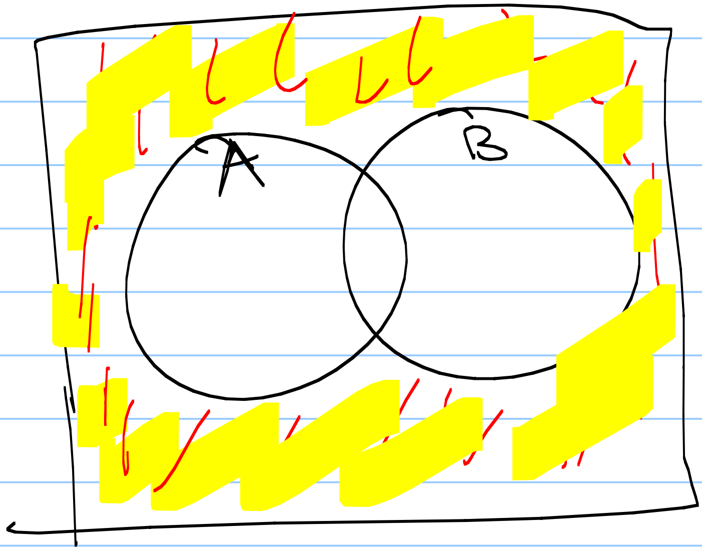
$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$

$$\overline{A \cap B} = \overline{A} \cup \overline{B}$$

look familiar?

Logic version:

$$\neg(p \vee q) = \neg p \wedge \neg q$$



Prove that $\overline{A \cap B} = \overline{A} \cup \overline{B}$.

pf: How?

2 things:

$\overline{A \cap B} \subseteq \overline{A} \cup \overline{B}$

$\overline{A} \cup \overline{B} \subseteq \overline{A \cap B}$

Key idea: write logic statement & use logic version of De Morgan's law

Another example of direct proofs.

Notation:

$$(A_1 \cup A_2) \cup A_3 = A_1 \cup (A_2 \cup A_3)$$

We will write

$$\bigcup_{i=1}^{n} A_i = A_1 \cup A_2 \cup \cdots \cup A_n$$

$$\bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap \cdots \cap A_n$$

Tuples

In sets, order doesn't matter: $\{1,2\} = \{2,1\}$

Sometimes, order should matter!

A <u>tuple</u> is an ordered list of objects.

<u>Ex</u>: $(2,2,8) \neq (2,8)$

$(1,2) \neq (2,1)$

$()$

$(\emptyset, \{2\}, \{3,8\})$

A tuple with n entries is an <u>n-tuple</u>.
(If $n=2$, an <u>ordered pair</u>.)

Cartesian Product

Dfn: Given sets A & B, the product of A & B, written $A \times B$, is the set of ordered pairs where the first element is from A and the second element is from B.

$$A \times B = \{ (a,b) \mid a \in A \wedge b \in B \}$$

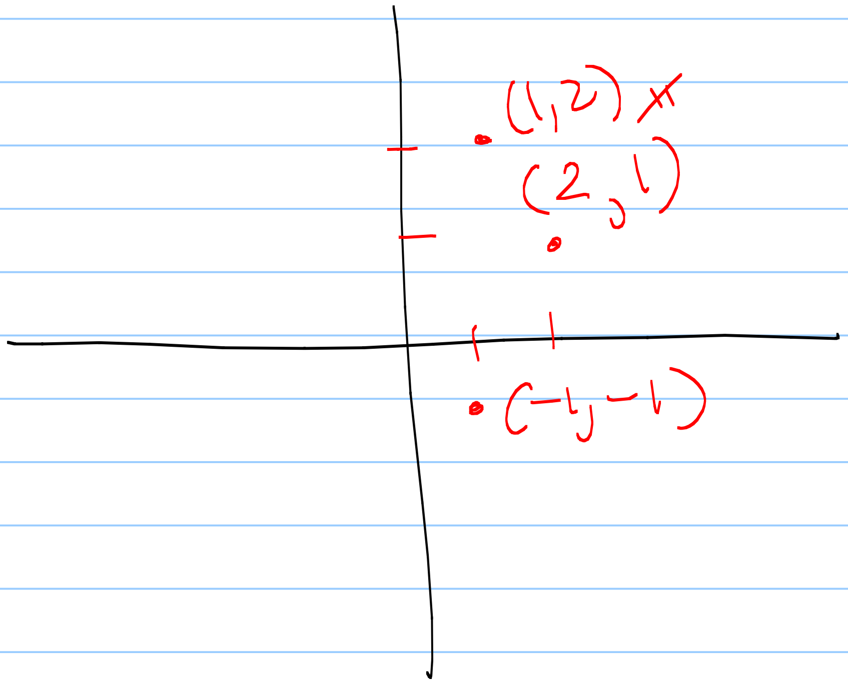
Ex: $A = \{a, b, c\}$ $B = \{1, 2\}$

$A \times B = \{ (a,1), (b,1), (c,1), (a,2), (b,2), (c,2) \}$

$(1,a) \notin A \times B$

$(1,1) \notin A \times B$

<u>Another</u>: $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$



With more than 2 sets, have:

$$A_1 \times A_2 \times \cdots \times A_n = \{ (a_1, a_2, \ldots, a_n) \mid \forall i,\ a_i \in A_i \}$$

Notation: $A^n = A \times A \times \cdots \times A$

So $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$

$\mathbb{R}^3 = \mathbb{R} \times \mathbb{R} \times \mathbb{R}$

$\vdots$

ie: $a \in A$, $b \in B$, $c \in C$

Caution: $(A \times B) \times C \neq A \times B \times C$

Typical element in $(A \times B) \times C$: $((a,b),c)$

But in $A \times B \times C$: (a,b,c)

Another: What is $\emptyset \times \{a,b\}$?

$= \{(x,y) \mid x \in \emptyset \text{ and } y \in \{a,b\}\}$

$= \emptyset$