Note Title 11/26/2012

Math 135 – More probability

Announcements

→ Midterm 2 graded
Average: ~ ~~5~~3.3..
Grade out of 66

– HW due ~~Wed~~ Fri.

– Final HW due last of class

– Final exam: Wed the 12th at noon

$\underline{2}\ \underline{2}\ \underline{1}$

Review from worksheet 11

(16) How many palindromes of length n?
(binary strings)

Rule of product: $\underbrace{\underline{2}\ \underline{2}}\ \underline{2}\ \underline{\ }\ \underline{\ }\ \underline{\ }\ \underline{1}\ \underline{1}\ \underline{1}$

n total

$$\text{total} = 2^{\lceil n/2 \rceil}$$

(12) A group has n men & n women. How many ways to arrange them if they must alternate?

Rule of product

$$\underline{2n}\ \underline{n}\ \underline{(n-1)}\ \underline{(n-1)}\ \underline{(n-2)}\ \underline{(n-2)} \cdots \underline{\ \ }\ \underline{\ \ }$$

2n spots

total: $2(n!)(n!)$

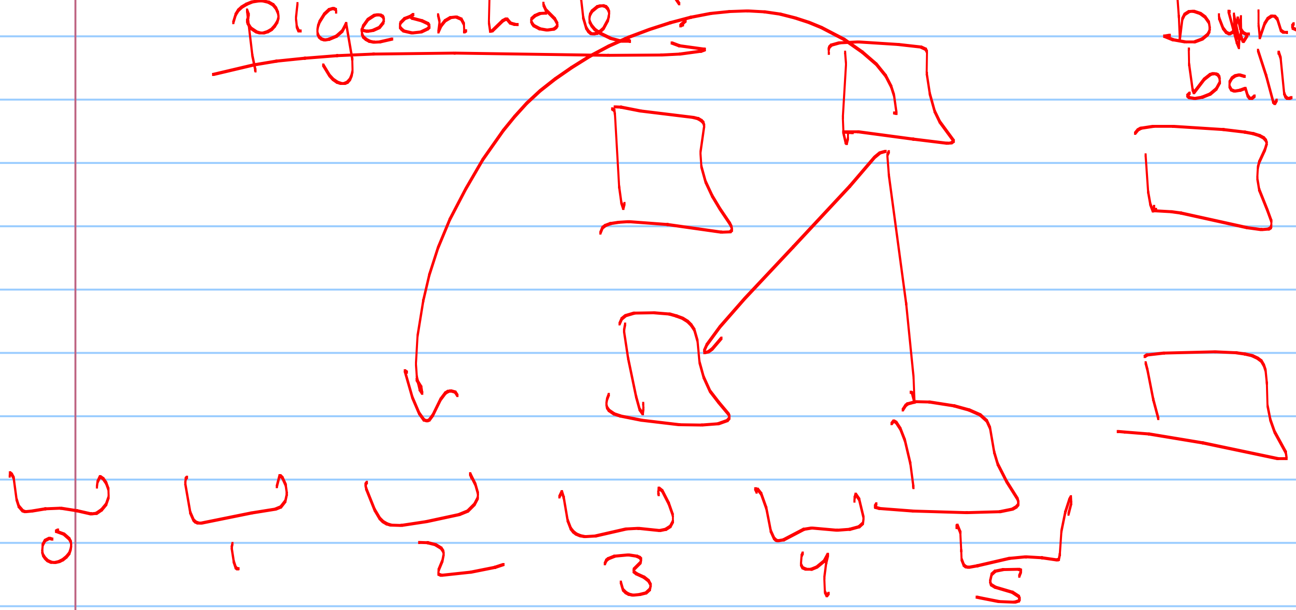
Another way: $n!$ ways to arrange men
$n!$ way " " women
2 choices of who starts our line

② A newtwork has 6 computers, each connected to 0 or more others. Show that at least 2 are connected to same # of others.

pigeonhole:

bins: # of computers each connects to

balls: computers



6 bins, but only 5 can be used (since 0 + 5 can't both be full)

Permutations, since order matters

$\frac{100}{1^{st}}\ \frac{100}{2^{nd}}\ \frac{100}{3^{rd}}\ \frac{100}{4^{th}}$

3. 100 tickets sold to different people.
4 prizes - all different.
How many ways to award if:

- no restrictions? 100^4 or $\underbrace{100 \cdot 99 \cdot 98 \cdot 96}_{\text{intent}}$
- person holding 47 wins grand prize? $99 \cdot 98 \cdot 97$
- person holding 47 doesn't win? $99 \cdot 98 \cdot 97 \cdot 96$
- both 19 & 47 win a prize? $4 \cdot 3 \cdot 98 \cdot 97$
- grand prize winner has 18, 47, 73, or 97? $\underline{4} \cdot \underline{99} \cdot \underline{98} \cdot \underline{97}$

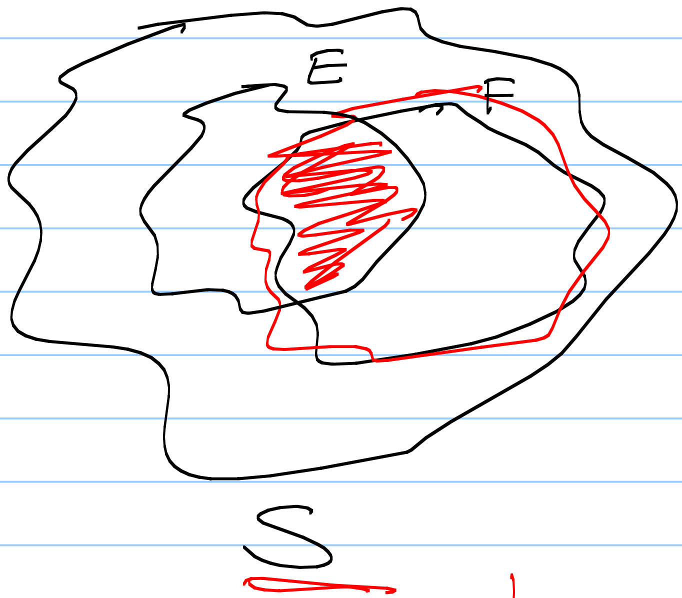
(4) Show that $k\binom{n}{k} = n\binom{n-1}{k-1}$ using comb. pf.

Count # ways to choose a committee with / a chairman.

LHS: Choose committee of size k: $\binom{n}{k}$
(rule of product)
then elect a chair out of k choices

RHS: Pick 1 chairman out of n people
Then choose k-1 other members of committee.

Conditional Probability



Let E and F be events with $p(F) > 0$.

The conditional probability of E given F, $p(E|F)$, is defined as

$$p(E|F) = \frac{p(E \cap F)}{p(F)} = \frac{|E \cap F|}{|F|}$$

know F is true

Q: What is conditional probability that a family with 2 children has 2 boys, given that they have at least 1 boy?

(Assume girls & boys equally likely.)

$S = \{GG, GB, BG, BB\}$

E = family has 2 boys

F = at least one boy

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{1/4}{3/4} = 1/3$$

Bayes Theorem

Conditional probability can prove useful!

Ex: Suppose we are testing for a disease with a test that has some accuracy.
A person who tests positive would want to know the odds that they actually have it, or perhaps would want to know how many times they should retake the test to be confident.

Thm (Bayes) Spps E & F are 2 events from a sample space with $p(E) \neq 0$ and $p(F) \neq 0$.

Then

$$p(F|E) = \frac{p(E|F)\,p(F)}{p(E|F)\,p(F) + p(E|\overline{F})\,p(\overline{F})}$$

Idea: $p(F|E) = \dfrac{p(E \cap F)}{p(E)}$

Ex (simpler): We have 2 boxes.
First has 2 green balls & 7 red.
Second has 4 green & 3 red.
Bob selects a box & then a random ball.
If the ball is red, what is the probability it came from the 1st box?

F = ball came out of 1st box
E = ball is red

$$p(F|E) = \frac{p(E|F)\,p(F)}{p(E|F)\,p(F) + p(E|\overline{F})\,p(\overline{F})}$$

$$= \frac{(7/9)\left(\frac{1}{2}\right)}{(7/9)\left(\frac{1}{2}\right) + (3/7)\left(\frac{1}{2}\right)} \approx .645\ldots$$

Ex: Spps one person in 100,000 has a rare disease. The test is accurate 99% of the time for someone with the disease, and correct 99.5% of the time for someone without it.

What is the probability that someone who tests positive actually has the disease?

F = probability he has disease

E = test returned positive

$$p(F|E) = \frac{p(E|F)\,p(F)}{p(E|F)\,p(F) + p(E|\overline{F})\,p(\overline{F})}$$

test positive

have disease

$$p(E|F) = .99$$

$$p(F) = \frac{1}{100,000} = .00001$$

$$p(\bar{F}) = \frac{99,999}{100,000}$$

$$p(E|\bar{F}) = .005$$

$$p(F|E) = \frac{(.99)(.00001)}{(.99)(.00001) + (.005)(.99999)}$$

$$\approx .002$$

Only .2% chance you actually have disease.