Note Title

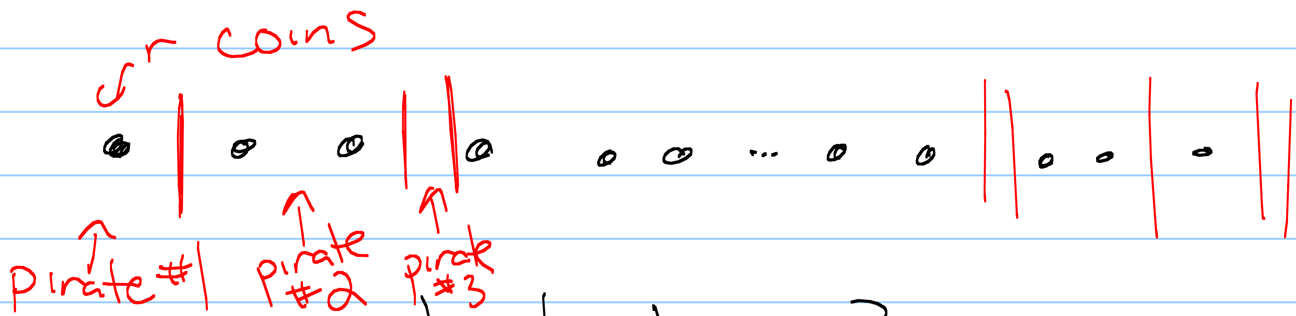
11/16/2012

Math 135 - Discrete Probability

Combinations revisited

How many ways are there to distribute r identical gold coins among n pirates?

Trick: Place coins in a row:



How can we divide them?

put n-1 dividers in

In total, have $\underbrace{r}_{\text{coins}} + \underbrace{(n-1)}_{\text{bars}}$

(so n piles)

Need to choose r spaces for the coins – rest will be bars

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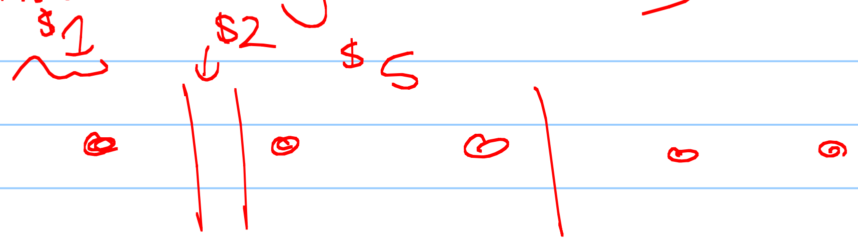
r coins
$n-1$ dividers

$$\binom{r+n-1}{r}$$

Q: How many ways are there to select 5 bills from a cash drawer containing \$1, \$2, \$5, \$10, \$20, \$50, and \$100 bills?

Note: Bills of same type are indistinguishable and we have at least 5 of each type.

$r = 5$ bills → gold coins



piles or "pirates" – types of bills

$n = 7$ piles

$$\text{ans} = \binom{5+7-1}{5} = \binom{11}{5}$$

Q: Suppose a cookie shop has 4 different kinds of cookies.
How many different ways to choose 6?

"coins" are 6 cookies
"pirates" are 4 categories

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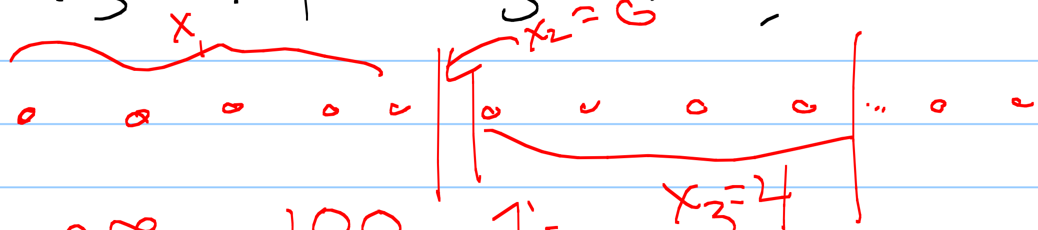
$r = 6$
$n = 4$

$$\binom{r+n-1}{r} = \binom{6+4-1}{6} = \binom{9}{6}$$

Q: How many non-negative integer solutions are there to:

$$x_1 + x_2 + x_3 + x_4 + x_5 = 100?$$



"coins" are 100 1's

$r = 100$

5 piles, so $n = 5$

$$\binom{100+5-1}{100}$$

Binomial Theorem

$$(x+y)^n = \sum_{i=0}^{n} \binom{n}{i} x^{n-i} y^i$$

$(x+y)^{17}$ what is coeff of $x^{12}y^5$?

$$\therefore \binom{17}{5}$$

Definitions : Probability

- An experiment is a procedure that yields one of a given set of outcomes.
- The sample space S is the set of possible outcomes.
- An event E is a subset of the sample space.

The probability of E if all events are equally likely is: $p(E) = \frac{|E|}{|S|}$

Example: A bowl has 4 blue balls and 5 red balls.
What is the probability that a ball chosen is blue?

$$\frac{4}{9}$$

Ex: Suppose 2 six-sided dice are rolled. What is the probability the sum is 7?

S = all possible outcomes

Rule of product: $6 \cdot 6 = 36 = |S|$

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$$\left.\begin{array}{l} 1,6 \\ 2,5 \\ 3,4 \\ 4,3 \\ 5,2 \\ 6,1 \end{array}\right\} \quad |E| = 6$$

$$p(E) = \frac{6}{36} = \frac{1}{6}$$

Rule of product: $\underline{10} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10}$

Ex: Consider a lottery where you pick 4 numbers between 0 and 9.

To win the big prize, must match all in correct order.

$$P(\text{winning}) = \frac{1}{10{,}000}$$

To win small prize, must match 3 of the 4.

$$\frac{4 \cdot 9}{10{,}000}$$

$\underline{1} \cdot \underline{1} \cdot \underline{1} \cdot \underline{9}$ +

$\underline{1} \cdot \underline{1} \cdot \underline{9} \cdot \underline{1}$ +

$\underline{\ } \ \underline{9} \ \underline{\ } \ \underline{\ }$ +

$\underline{9} \ \underline{\ } \ \underline{\ } \ \underline{\ }$

Q: What is the probability of getting 4 of a kind in poker?

$$|S| = \binom{52}{5}$$ (order of cards doesn't matter)

How many ways to get 4 of a kind?

$$\binom{13}{1} \cdot \binom{48}{1}$$

# of ways to choose a "kind"

Q: What is the probability of getting a full house (2 of a kind + 3 of a kind) in a poker hand?

$$\frac{\binom{13}{1}\binom{4}{2}\binom{12}{1}\binom{4}{3}}{\binom{52}{5}}$$

Q: A sequence of 10 bits is generated randomly.
What is the probability that at least one bit is 0?

(Trick - count the opposite!)

$1 - P(\text{all } 1\text{'s})$

$= 1 - \frac{1}{2^{10}}$

Q: What is the probability that a random integer between 1 and 100 is divisible by 2 or 5?

When all outcomes are not equally likely, things are more complex.

S = sample space

$s \in S$ is a possible outcome

Each $s \in S$ gets probability $p(s)$, where:

① $0 \le p(s) \le 1$

② $\sum_{s \in S} p(s) = 1$

p is called a probability distribution.

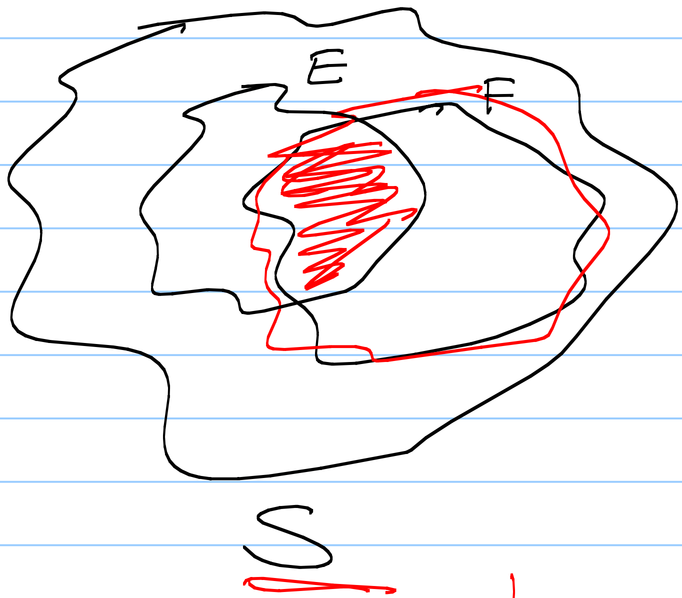
Probability of an event

If $E \subseteq S$, then

$$p(E) = \sum_{s \in E} p(s)$$

Note: $p(E) + p(\overline{E}) = 1$

Conditional Probability



Let E and F be events with $p(F) > 0$.

The conditional probability of E given F, $p(E|F)$, is defined as

$$p(E|F) = \frac{p(E \cap F)}{p(F)} = \frac{|E \cap F|}{|F|}$$

know F is true

$2 \cdot 2 \cdot 2 \cdot 2$

Ex: A coin is flipped 4 times, where $p(H) = p(T) = \frac{1}{2}$.

What is the probability it has 2 consecutive tails (E), given first flip results in heads?

F = flip results in heads

$P(F) = \frac{8}{16} = \frac{1}{2}$

Prob. that has 2 consecutive T's and first flip was H $= \frac{3}{16}$

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Ans: $\frac{3}{16} \div \frac{1}{2} = \frac{6}{16} = \boxed{\frac{3}{8}}$

Q: What is conditional probability that a family with 2 children has 2 boys, given that they have at least 1 boy?

(Assume girls & boys equally likely.)