Math 135 - Logic & Predicates

Note Title 8/29/2012

Announcements

- HW0 due Friday at start of class

- Office hours - see webpage

A few tools

Summations:

$$\sum_{i=1}^{6} 1 = 6$$

$$\sum_{i=0}^{n} 1 = n+1$$

$$\sum_{i=1}^{n} 1 = \overbrace{(1+1+1+\cdots+1)}^{n} = n$$

$$\sum_{i=1}^{n} i = (1+2+3+\cdots+(n-1)+n) = \frac{n(n+1)}{2}$$

$$\rightarrow \sum_{i=1}^{n} c^i = \frac{c^{n+1}-c^0}{c-1} \quad (?)$$

c: constant

$\frac{c^{n+1}-c^0}{c-1}$: in book

Mod - remainders

%

$11 \bmod 12 = 11$

$13 \bmod 12 = 1$

$25 \bmod 12 = 1$

take remainder

$$12\overline{)11}\quad 0,\ 11$$

$$12\overline{)25}\quad 2,\ 24,\ 1$$

Logarithms

$x^2 \cdot x^3 = x^5$

What is $\log_2 2$? $= 1$

$\log_5 25$? $= 2$

$(x^6)^2 = x^{12}$

Rules:

$$\log_x (ab) = \log_x a + \log_x b$$

$$\log_x (y^a) = a \log_x y$$

$$\frac{\log_x a}{\log_x b} = \log_b a$$

Logical Equivalence

Propositions that have the same truth values (in whole truth table) are called logically equivalent.

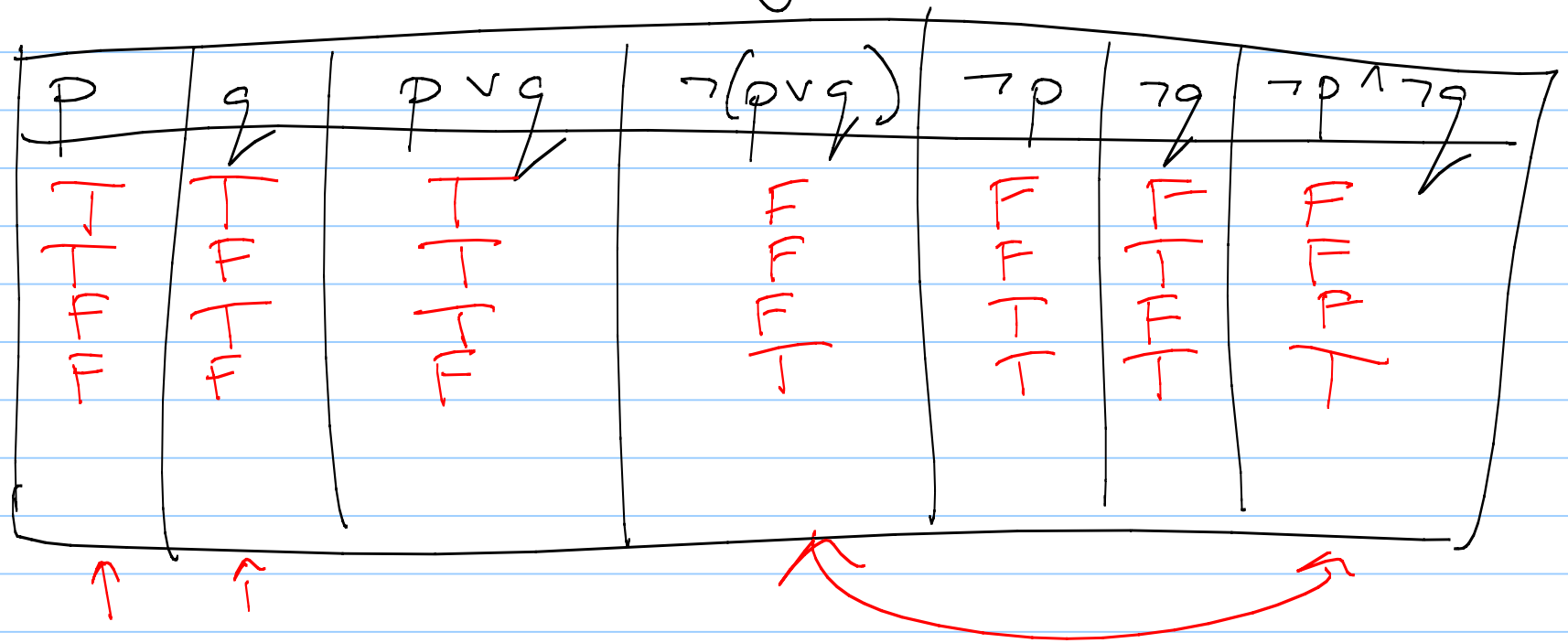
(written $p \equiv q$)

Ex: implication $(p \to q)$

and its contrapositive $(\neg q \to \neg p)$

Example 2: $\neg(p \vee q) \equiv \neg p \wedge \neg q$

Why? (DeMorgan's Law)



p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

Predicates $P(x)$

Propositions that depend on a variable

Ex: $P(x): x > 0$

$Q(x,y): x + y = 3$

$R(x)$: "x is in discrete math"

$S(x)$: "x is a prime number"

$T(x)$: "x is a SLU student"

Note:
- Can combine theses
- Truth value depends on variable

Ex: $P(2)$: True

$Q(1, 11)$: False

$R(x) \wedge T(x)$: true if x is a student in this room

$\neg S(6)$: True

Application: Truth tellers + liars

Suppose we meet 2 people, Alice + Bob.

Alice says: "Exactly one of us is lying."

Bob says: "At least one of us is telling the truth."

Who is telling the truth?

Let p = "Alice is truthful."

q = "Bob is truthful."