Math 135 - Lecture 1

Note Title 8/27/2012

Announcements

- Syllabus posted
- HW0 is due Friday

Logic

Dfn: A proposition is a declarative statement which is either true or false, but not both.

Ex: This is math 135.

I am a teacher.

Ambiguous: Spiders are creepy.

Math is hard.

Negation

Let p be a proposition.

The negation of p, written $\neg p$, is the statement:

"It is not the case that p."

Ex: p = "The sky is yellow."

$\neg p$ = "The sky is not yellow."

(Note: English gives many ways to word - just be careful!)

Conjunction + Disjunction
(ie "and" and "or")

AND

Conjunction: "p and q", written $p \wedge q$, is true exactly when both p and q are (both) true (and is false otherwise)

OR

Disjunction: "p or q", written $p \vee q$, is true iff either of p, q is true, or if both are true

Truth Tables

p	q	$p \vee q$ (OR)	$p \wedge q$ (AND)	$\neg(p \vee q)$
T	T	T	T	F
T	F	T	F	F
F	T	T	F	F
F	F	F	F	T

Exclusive or: $p \oplus q$

True when exactly one of p, q is true (but not both)

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Implications $\quad p \to q$

"If p, then q"

"p implies q"

"q if p"

Ex: "$\overbrace{\text{If I am elected}}^{p}$, then I $\underbrace{\text{will lower taxes}}_{q}$."

When is it true?

Dfn:

$p \to q$ is false when p is true and q is false (and if q is true otherwise)

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

vacuously true

Examples to ponder:

① "If $\overbrace{\text{today is Friday}}^{p}$, then $\overbrace{2+5=7}^{q}$."

True! F T

"If $\underbrace{\text{today is Friday}}_{p \;\; F}$, then $\underbrace{2+1=4}_{F \;\; q}$."

true

Converse, Inverse & Contrapositive

Consider $p \to q$.

The converse is $q \to p$.

The inverse is $\neg p \to \neg q$.

The contrapositive is $\neg q \to \neg p$.

Ex:

If the number 15 is prime, then it has no divisors.

$p \to q$

T or F?

F (p) = 15 is prime

F q = 15 has no divisors.

$q \to p$: If 15 has no divisors, then 15 is prime

$\neg p \to \neg q$: If 15 is not prime, then 15 has divisors.

$\neg q \to \neg p$: If 15 has divisors, then 15 is not prime.

Ex: Truth Tables!

p	q	$\neg p$	$\neg q$	$p \to q$	$q \to p$	$\neg p \to \neg q$	$\neg q \to \neg p$
T	T	F	F	T	T	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T

Logical Equivalence

Propositions that have the same truth values (in whole truth table) are called logically equivalent.

(written $p \equiv q$)

Ex: