Math 135 - More Induction

Note Title 10/12/2012

Announcements

- HW is due
- Next HW is over induction

Induction − Ch 5 (?)

A proof technique that is used to prove propositions of the form:

$$\forall n,\ P(n)$$

$\forall n \geq c$

Idea: ① Show $P(1)$ is true

② Show $\forall k > 1,\ P(k-1) \rightarrow P(k)$

Since $P(1)$ is true (by ①),

$P(1) \rightarrow P(2)$ (by ②)

$P(2) \rightarrow P(3)$ (by ②)

$\vdots$

How to write inductive proofs

3 required parts

Base case : Show $P(1)$ is true.

Inductive Hypothesis : Assume $P(k-1)$ is true

Inductive Step : Show $P(k)$ is true.

$$\underbrace{\text{Ex:}}\quad \overset{P(4)}{\underline{\forall n \geq 4}},\ \overbrace{2^n < n!}^{P(n)}$$

pf: induction on n

Base case: $n=4$

$2^4 = 16$ $\quad 4! = 4\cdot3\cdot2\cdot1 = 24$

$16 < 24$ ✓

IH: Assume $2^{n-1} < (n-1)!$

IS: $2^n = 2\cdot 2^{n-1}$

$< 2\cdot(n-1)!$ by IH

and since know $n > 2$ $\quad < n\cdot(n-1)! = n!$

$\Rightarrow 2^n < n!$ ∎

Geometric Series:

$$\sum_{i=0}^{n} a \cdot r^i = \frac{ar^{n+1} - a}{r-1}$$

pf: induction on n

Base Case: $n=0$

$$\sum_{i=0}^{0} a \cdot r^i = a \cdot r^0 = a$$

$$\frac{a \cdot r^{0+1} - a}{r-1} = \frac{ar - a}{r-1} = \frac{a\cancel{(r-1)}}{\cancel{r-1}} = a \quad \checkmark$$

Assume:

IH: $\sum_{i=0}^{n-1} a \cdot r^i = \frac{ar^n - a}{r-1}$

IS: $\sum_{i=0}^{n} a \cdot r^i = \sum_{i=0}^{n-1} a \cdot r^i + a \cdot r^n$

use IH

$= \frac{ar^n - a}{r-1} + a \cdot r^n$

$= \frac{ar^n - a + a \cdot r^n (r-1)}{r-1} = \frac{a \cdot r^n - a + ar^{n+1} - ar^n}{r-1}$

$= \frac{ar^{n+1} - a}{r-1}$ $\boxed{}$

Ex: If n is a positive integer, then $n^3 - n$ is divisible by 3.

Base case: $n = 1$

Consider $1^3 - 1 = 0$ ✓

divisible by 3

IH: Assume $k^3 - k$ is divisible by 3

IS: Consider $(k+1)^3 - (k+1) =$

$(k+1)(k^2+2k+1) - (k+1)$

$= k^3 + 2k^2 + k + k^2 + 2k + 1 - (k+1)$

$= k^3 + 3k^2 + 2k$

$= \underbrace{k^3 - k}_{\text{IH} \Rightarrow \text{div by } 3} + \underbrace{3k^2 + 3k}_{\text{div by } 3} \Rightarrow$ sum is div by 3 $\boxtimes$