Math 135 - Induction

Note Title 10/10/2012

Induction - Ch 5 (?)

A proof technique that is used to prove propositions of the form:

$\forall n,\ P(n)$

$\forall n \geq c$

Idea: ① Show $P(1)$ is true

② Show $\forall k > 1,\ P(k-1) \rightarrow P(k)$

Since $P(1)$ is true (by ①),

$P(1) \rightarrow P(2)$ (by ②)

$P(2) \rightarrow P(3)$ (by ②)

$\vdots$

Example: $\forall n \geq 1$, $\underbrace{\sum_{i=1}^{n} i}_{LHS} = \underbrace{\frac{n(n+1)}{2}}_{RHS}$

pf: by induction on n

① Show $P(1)$ is true

$\boxed{n=1}$ $\underbrace{\sum_{i=1}^{1} i}_{LHS} = 1$ RHS: $\frac{1(1+1)}{2} = 1$

② $\forall k > 1$, $P(k-1) \rightarrow P(k)$

Assume $\left.\sum_{i=1}^{k-1} i = \underbrace{\frac{(k-1)((k-1)+1)}{2}}\right\} P(k-1)$

Consider $\sum_{i=1}^{k} i = \sum_{i=1}^{k-1} i + k = \frac{(k-1)(k)}{2} + k$

$P(k-1) \rightarrow P(k)$!

Consider $\displaystyle\sum_{i=1}^{k} i = \sum_{i=1}^{k-1} i + k = \frac{(k-1)(k)}{2} + k$

$$= k\left(\frac{(k-1)}{2} + 1\right)$$

$$= k\left(\frac{(k-1)+2}{2}\right) = \overbrace{k\left(\frac{k+1}{2}\right)}^{P(k)}$$

How to write inductive proofs

3 required parts

Base case : Show $P(1)$ is true.

Inductive Hypothesis : Assume $P(k-1)$ is true

Inductive Step : Show $P(k)$ is true.

Ex: Show that the sum of the first n odd integers is n^2.

$$\sum_{i=1}^{n} (2i-1) = n^2$$

pf: by induction on n

Base case: Let n=1.

$\sum_{i=1}^{1} (2i-1) = 1$ $\quad$ $n^2 = 1^2 = 1$

they are equal.

Ind. Hyp: Assume $\sum_{i=1}^{n-1}(2i-1) = (n-1)^2$

Ind. Step: Consider $\sum_{i=1}^{n}(2i-1) = \sum_{i=1}^{n-1}(2i-1) + (2n-1)$

use IH

$= (n-1)^2 + (2n-1) = n^2 - 2n + 1 + (2n-1)$

$= n^2$ $\boxtimes$

$x = y < z < a$
$\Rightarrow x < a$

Ex: $\forall n > 0, \ n < 2^n$

pf: by induction on n

Base case: $n = 1$

$n = 1 \qquad 2^n = 2^1 = 2$

$1 < 2$ ✓

IH: Assume $n-1 < 2^{n-1}$

use IH

IS: $n = (n-1) + 1 < 2^{n-1} + 1$

since $n > 1$ $\qquad < 2^{n-1} + 2^{n-1}$

$= 2 \cdot 2^{n-1} = 2^n$ ∎