More on Graphs

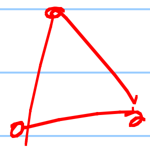
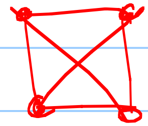
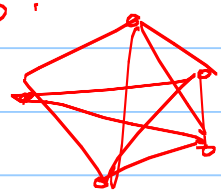
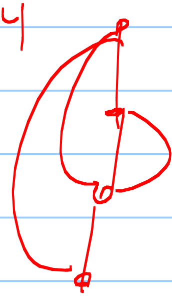
Note Title 11/29/2012

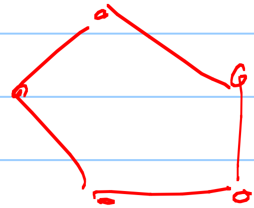
Announcements

- HW is due now
- Next HW - posted soon
- Review is last day of class
  Final is Wed @ noon

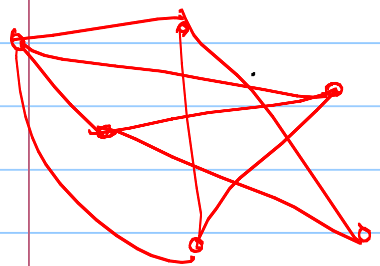
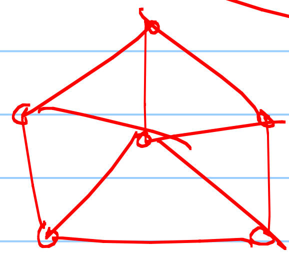
Some Special Graph Classes

K_n: the complete graph on n vertices

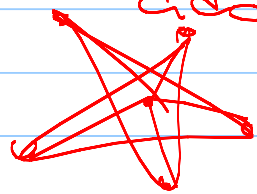
K_3  K_4  K_5:  K_4 

C_n: the n-cycle C_5 

and
W_n : Wheels

 W_5 

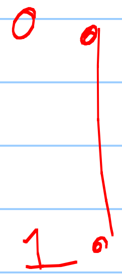
take C_n & add a "center"



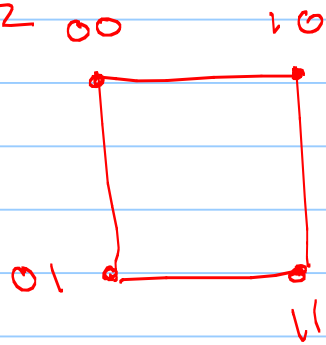
Hypercubes Q_n:
vertices for each bit string of length n
2 vertices are adjacent if they differ in exactly 1 bit.

Ex:

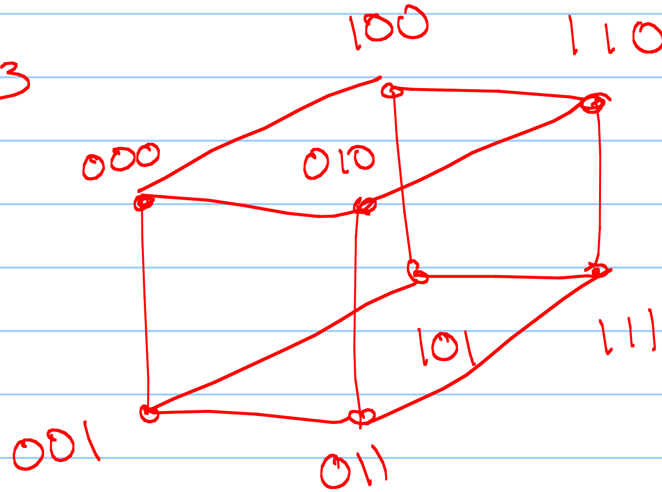
Q_1



Q_2

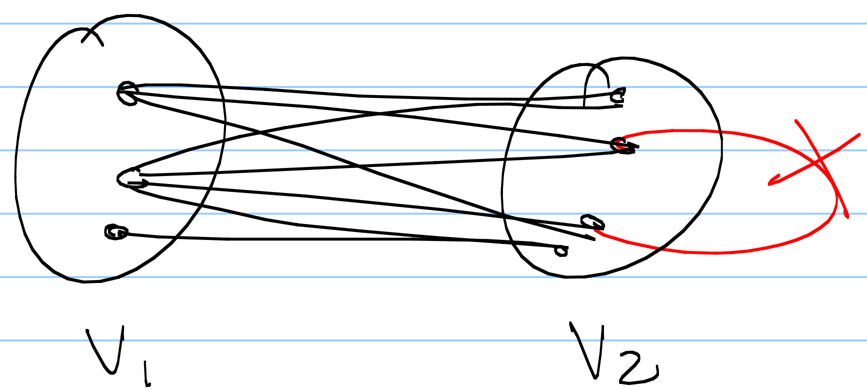


Q_3



Biparitite Graphs

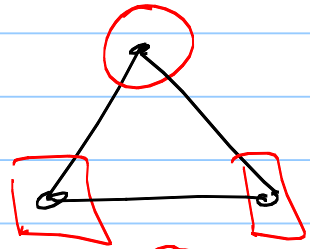
A simple graph is bipartite if its vertex set can be partitioned into 2 disjoint sets V_1 + V_2 so that all edges go between vertices in the different sets.



Examples: Are they bipartite?

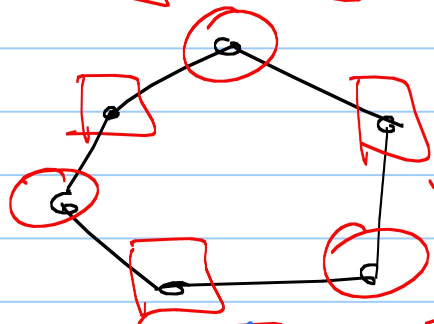
Circles & squares

K_3



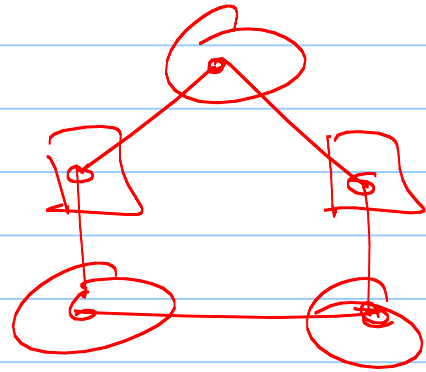
Not bipartite

C_6 :



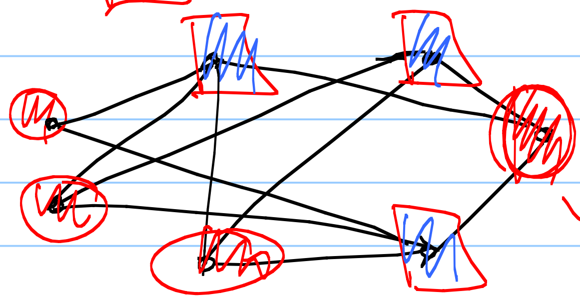
Yes

C_5



No

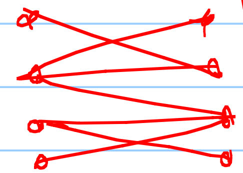
G :

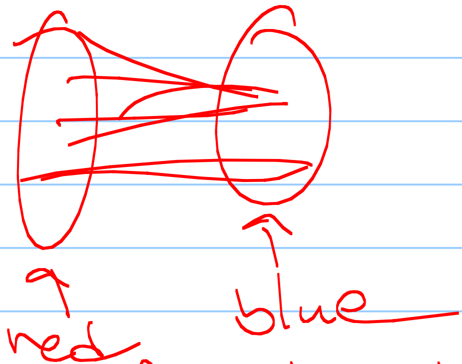


Yes

Thm: A graph is bipartite if & only if we can color each vertex one of two colors so that no 2 adjacent vertices have same color.

pf: $\Rightarrow$: Say G is bipartite.

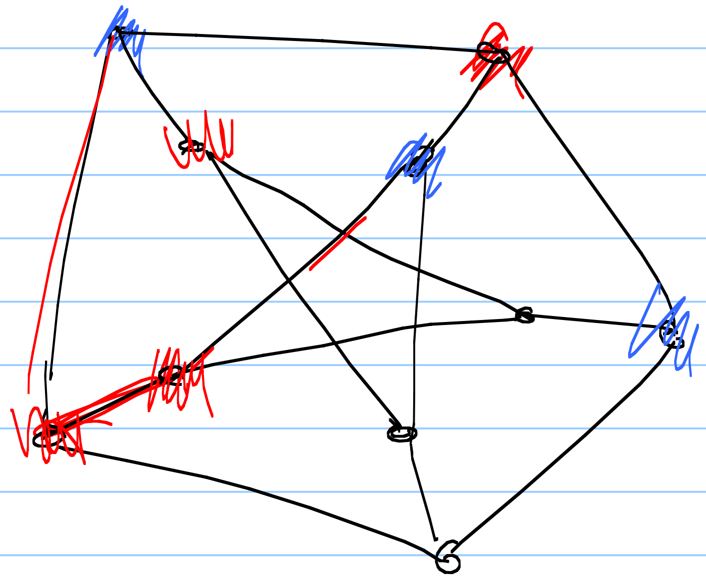




$\Leftarrow$: Color vertices so every edge goes b/t different colors.
red vertices $\to$ set 1
blue vertices $\to$ set 2
only edges go between the sets.

Now use this!

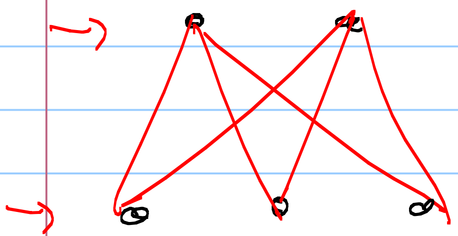
Biportite?



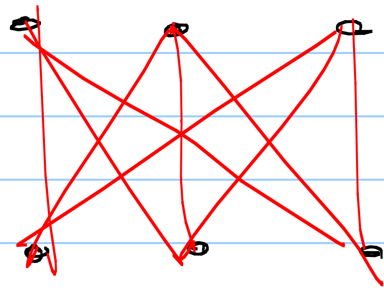
No

The complete bipartite graph $K_{m,n}$ is a bipartite graph with two vertex sets of size m & n with all possible edges.

$K_{2,3}$



$K_{3,3}$

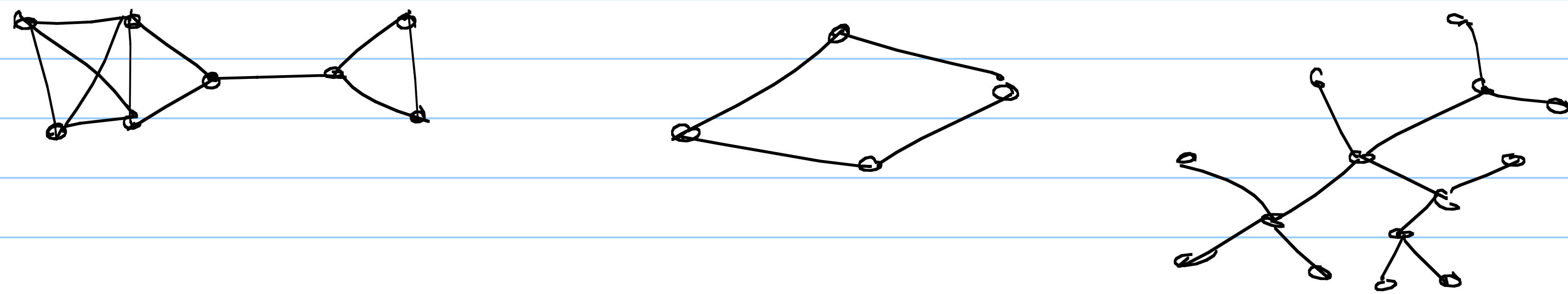


Lemma: Every u-v walk contains a u-v-path.

[Cor: In a connected graph, every pair of vertices has a simple path between them.]

Dfn: A cut edge in a graph is an edge whose deletion increases the number of components.

A cut vertex is a vertex whose deletion increases # of components.



Thm: An edge is a cut edge $\iff$ it does not belong to any cycle.

Last time: Eulerian circuit

An Eulerian circuit uses every edge in a graph exactly once.

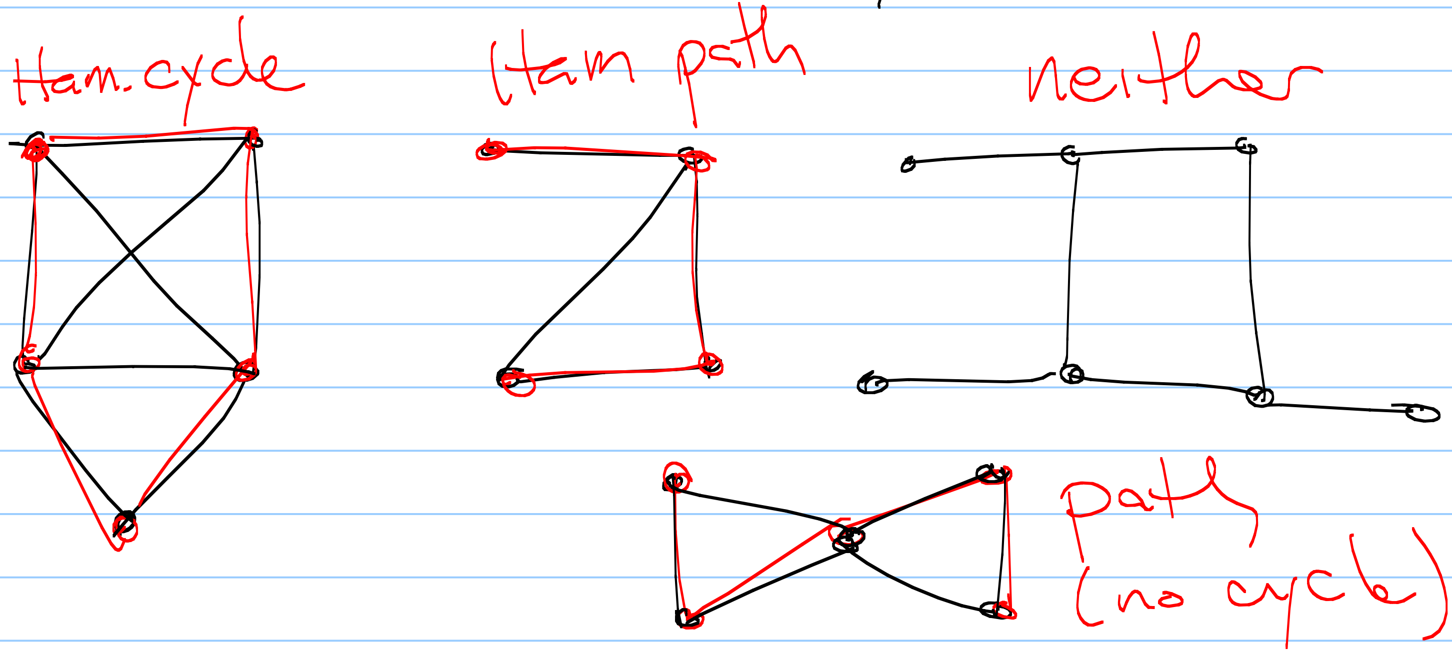
Simple characterization:

A graph has an Euler circuit $\Longleftrightarrow$ G is connected & all vertices have even degree.

Hamiltonian Paths

A related concept is a Hamiltonian path (or cycle), which visits each vertex exactly once.

Ex:



There is no easy way to check if a graph G has a Hamiltonian cycle.

Some results:

Dirac's Theorem:

If G is a simple graph with n vertices where every vertex has degree $\geq n/2$, then G has a Hamiltonian circuit.

This problem is NP-Hard.