Math 135: Discrete Mathematics, Fall 2012
Homework 9

Due *in class* on Friday, Nov. 28

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1. (a) The name of a variable in the C programming language is a string that can contain uppercase letters, lowercase letters, digits, or underscores. Further, the first character much be a letter (either upper or lower case) or an underscore. If the name of a variable is determined only by its first 8 characters (the rest are ignored), how many different variables can be named in C?

   (b) How many different ways are there to choose 8 donuts from the 17 varieties at a donut shop?

   (c) How many solutions are there to the inequality $x_1 + x_2 + x_3 + x_4 \leq 25$, where each x_i is a nonnegative integer?

   (d) A bowl contains 20 red balls and 20 blue balls. A woman selects balls at random without looking at them. How many balls must she select in order to be sure of having at least 4 balls of the same color?

   (e) How many bit strings of length either start with 3 consecutive 1's or end with 5 consecutive 0's?

   (f) What is the coefficient of x^9y^9 in the expansion of $(2x - y)^{18}$? (Hint: Go look up the binomial theorem again!)

2. Give a combinatorial proof of the following identity:

   $$\binom{n}{r}\binom{r}{k} = \binom{n}{k}\binom{n-k}{r-k}$$

3. Show that if $n + 1$ integers are chosen from the set $\{1, 2, \ldots, 3n\}$, then there are always two which differ by at most 2.

4. (a) Which is more likely: rolling a total of 8 when 2 6-sided dice are rolled, or rolling a total of 8 when 3 dice are rolled? Justify your answer.

   (b) Suppose that 100 people enter a contest where winners are selected with equal probability, no one can win more than 1 prize, and there are 3 different prizes offered. What is the probability that Grace, Edwin and Maggie each win one of the prizes?

   (c) Suppose we generate a random permutation of the alphabet. What is the probability that F is in its original position in the permutation, assuming all permutations are equally likely?

   (d) Eight women and fifteen men are on the faculty of a math department. If we are forming a committee of 6 members of the department, what is the probability that there is at least one woman and on the committee?

(e) What is the conditional probability that exactly four heads appear when a fair coin is flipped six times, given that the first flip comes up tails?

5. (a) Pascal's identity states that:

$$\binom{n+1}{r} = \binom{n}{r-1} + \binom{n}{r}$$

Prove this identity using the algebraic formula for $\binom{n}{r}$.

(b) Extra credit (for part b only): Prove the following identity holds for any $n \geq k$ via an induction proof:

$$\sum_{i=k}^{n} \binom{i}{k} = \binom{n+1}{k+1}$$

[Hint: In your inductive step, you'll need to use Pascal's identity from part a.]