Math 135: Discrete Mathematics, Fall 2012
Homework 6

Due *in class* on Friday, Oct. 19

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1. Use induction to prove that $n^n \geq n!$ whenever $n \geq 1$ is an integer.

2. Use induction to prove that

$$\sum_{k=1}^{n} k^2 = n(n+1)(2n+1)/6.$$

3. Use induction to prove that

$$\sum_{k=1}^{n} k \cdot k! = (n+1)! - 1.$$

4. Prove that for every positive integer n,

$$\sum_{k=1}^{n} k2^k = (n-1)2^{n+1} + 2$$

5. If A and B are sets, then the *symmetric difference* of A and B is written $A \triangle B$ and is defined to be $(A - B) \cup (B - A)$. Prove that if $A_1, A_2, \ldots, A_n$ are sets, then

$$(((A_1 \triangle A_2) \triangle A_3) \cdots \triangle A_{n-1}) \triangle A_n = \{x \ : \ x \text{ is in an odd number of the sets } A_1, A_2, \ldots, A_n\}.$$

6. Assume that a chocolate bar consists of n squares arranged in a rectangular pattern. The bar can only be broken along vertical or horizontal lines separating the squares. (Think of a Hershey's bar.)

   Assuming that only one piece can be broken at a time, determine how many breaks you must make in order to break the bar into n squares. Use induction to prove your answer is correct.