Math 135: Discrete Mathematics, Fall 2012
Homework 4

Due *in class* on Friday, Sept. 28, 2010

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1. Give an example of a function from $\mathbb{N}$ to $\mathbb{N}$ which is:

   (a) one-to-one but not onto
   (b) onto but not one-to-one
   (c) both onto and one-to-one (but NOT the identity function)
   (d) neither one-to-one or onto

2. For the following functions, decide whether each one is one-to-one, onto, and bijective, and prove each of your answers.

   (a) $f : \mathbb{R} \to \mathbb{R}$, with $f(x) = (x+1)/(x+2)$
   (b) $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$, with $f(m,n) = m + n + 1$
   (c) $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$, with $f(m,n) = m^2 - 4$

3. Let f be a function from the set A to the set B. Let S and T be subsets of A.

   (a) Prove that $f(S \cup T) = f(S) \cup f(T)$
   (b) Prove that $f(S \cap T) \subseteq f(S) \cap f(T)$
   (c) Give an example to show that the inclusion from part (b) may be proper - in other words, give examples of sets and a function where $f(S \cap T) \subset f(S) \cap f(T)$

4. Show that $x \log x$ is $O(x^2)$ but that x^2 is not $O(x \log x)$.

5. Give a big-O estimate (as tight as possible) of the following functions. Be sure to justify your answer, remembering that you are welcome to use theorems from class or the book.

   (a) $(n^3 + n^2 \log n)(\log n + 1) + (14 \log n + 11)(n^3 - n)$
   (b) $(2^n + n^2)(n^3 + 3^n)$
   (c) $f(x) = (x^2)(\pi + (16^{23!}))^7 + \sum_{i=0}^{10} \left(\frac{e}{i+1} + i\right)^{12} x^i$

6. Arrange the functions 2^n, $n^2 \log n$, $\sqrt{n}$, $(n!)^2$, and 10^n so that each function is big-O of the next one, and give a short proof for each pair to show that they are in the correct order. (You may use theorems or big-O bounds we proved in class or in the book if they are helpful.)

7. Extra credit: If f and $f \circ g$ are both one-to-one, does it follow that g is one-to-one? Justify your answer.