Math 135: Discrete Mathematics, Fall 2012
Homework 1

Due *in class* on Friday, Sept. 7, 2010

Submit your solutions for this homework *in class* on Friday, September 9. Please make sure to read the course policies on homework *before* writing up your homework.

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1. Write the negation, contrapositive, converse and inverse of the following statements.

   (a) If you do not like discrete math, then you will not like this homework.

   (b) There exists a positive integer n such that if n is odd, then $n^6 + n$ is odd.

   (c) If a triangle has either two equal sides or two equal angles, then it is an isosceles triangle.

2. Rewrite the following propositions as unambiguous English sentences, given the following prepositions (where the universe is all people).

   - $A(x)$ means "x likes Glee".
   - $B(x)$ means "x likes Buffy".
   - $C(x)$ means "x has good taste".
   - $D(x)$ means "x has netflix".

   For example the statements $\forall x[D(x) \to A(x)]$ could be translated to "For anyone, if you have netflix then you watch Glee" or (perhaps more succinctly) "Everyone who has netflix watches Glee.".

   (a) $\exists x[A(x) \wedge B(x) \wedge D(x)]$

   (b) $\forall x[B(x) \to D(x)]$

   (c) $\forall x[A(x) \vee B(x) \to C(x)]$

   (d) $\exists x[(A(x) \vee B(x)) \wedge C(x) \wedge D(x)]$

3. Express the **negations** of the following statements so that all negation symbols appear immediately preceding the predicates (and not outside any quantifiers or groups of predicates).

   (a) $\forall x \exists y[P(x, y) \to \neg Q(y, x)]$

   (b) $\exists x \exists y(\neg Q(x, y) \vee R(x, y))$

   (c) $\forall x \forall y \exists z \geq 0[(\neg A(x, y) \wedge B(x, y)) \to \neg C(x, y, z)]$

4. Classify the following formulas into logically equivalent groups. (Hint: Try using truth tables, or you can simplify using the logical equivalences in section 1.3 of the textbook, such as in Table 6.)

   (a) p

   (b) $p \vee q \vee \neg r$

   (c) $(\neg p) \rightarrow (q \rightarrow r)$

   (d) $(p \wedge q) \rightarrow p$

   (e) $p \wedge (q \vee \neg q)$

   (f) $q \rightarrow (p \vee r)$

   (g) $((p \rightarrow q) \wedge (p \rightarrow r))$

   (h) $p \rightarrow (q \wedge r)$

5. While walking across campus, you come across 3 people have an argument. The first, Alice, tells you, "Bob and Carol are both lying." The second, Bob, tells you, "Only one of the other two is lying". The third, Carol, tells you, "At least one of us is lying". Who, if anyone, is telling you the truth?

6. **Extra Credit:** Next, you come across two different people on your walk. Donald says, "I am lying if Erik is." Erik says, "Donald is lying if I am." Can you tell who if anyone is telling the truth?