Math 135: Discrete Mathematics, Fall 2010
Worksheet 8

1. Let f_n be the n^{th} Fibonacci number, defined as $f_0 = 0$, $f_1 = 1$, and $f_n = f_{n-1} + f_{n-2}$. Prove that $f_1 + f_3 + \ldots + f_{2n-1} = f_{2n}$, when n is a positive integer.

   Hint: Think induction!

2. Suppose n adjacent spaces are available for parking along a curb. We can fill the space using Rabbits, which are small and take only 1 space, or Cadillacs, which take 2 spaces. Write a recurrence for $P(n)$, the number of ways to fill n spaces with Rabbits and Cadillacs. Justify your answer!

3. Find a solution for the following recurrence, and prove your answer is correct using induction.

   $A(1) = 2$, and for all $n \geq 2$, $A(n) = A(n-1) + n - 1$

4. Find a recurrence for $b(n)$, the number of bitstrings of length n that do not have 3 consecutive zeroes. For example, $f(3) = 7$ because out of the 8 bitstrings of length 3 - $\{000, 001, 010, 011, 100, 101, 110, 111\}$ - only 1 has 3 consecutive zeros. (Remember your base cases also!)