Math 135: Discrete Mathematics, Fall 2010
Worksheet 6

1. Determine if each of these functions is $O(x)$, $\Omega(x)$, and $\Theta(x)$.

   (a) $f(x) = 10$

   (b) $f(x) = x^2 + x + 12$

   (c) $f(x) = 36x - 14$

   (d) $f(x) = \lfloor x/2 \rfloor$

2. Give as good a big-O estimate as possible for the following:

   (a) $(n^2 + 8)(n + 1)$

   (b) $(n \log n + n^2)(n^3 + 2)$

   (c) $(n! + 2^n)(n^3 + \log(n^2))$

3. Suppose that $f(x)$ is $O(g(x))$ and $g(x)$ is $O(h(x))$, and prove that $f(x)$ is $O(h(x))$.
   Hint: Use the definitions!

4. Show that the functions $f(n) = 2^{2\log_2 n}$ and $g(n) = 3n^2 + 14$ are asymptotically equivalent.

5. Find functions f and g from $\mathbb{N}$ to $\mathbb{R}^+$ such that $f(n)$ is not $O(g(n))$ and $g(n)$ is not $O(f(n))$.

6. Show that $\log n!$ is greater than $(n \log n)/4$ for $n > 4$.
   Hint: Begin with inequality $n! > n(n-1)(n-2)\cdots n/2$.