Math 135: Discrete Mathematics, Fall 2010 Worksheet 5

- 1. Determine if f is a function from the set of all bit strings (meaning strings where each digit is 0 or 1) to the set of integers for the following possible descriptions of f. If f is a function, also decide if it is 1-1 or onto (or both).
 - (a) f(S) is the position of a 0 bit in S.
 - (b) f(S) is the number of 1 bits in S.
 - (c) f(S) is the smallest integer i such that the i^{th} bit of the string S is 1, and f(S) = 0 when S is the empty string

2. (a) Prove that if $f : A \to B$ is onto and $g : B \to C$ is onto, then $g \circ f$ is onto.

(b) Prove that if both f and g are 1-1, then $g \circ f$ is 1-1.

- 3. Determine if the function $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ is onto and/or 1-1 if
 - (a) f(m,n) = m + n
 - (b) $f(m,n) = m^2 + n^2$
 - (c) f(m, n) = m

4. Let $f : \mathcal{P}(X) \to \mathcal{P}(Y)$ be a function such that for all $S, S' \in \mathcal{P}(X)$, $S \subseteq S'$ if and only if $f(S) \subseteq f(S')$. Prove that f is 1-1.

5. Let $f: X \to Y$ and $g: Y \to Z$ be bijections. Show that $(g \circ f)^{-1} = g^{-1} \circ f^{-1}$.