Math 135: Discrete Mathematics, Fall 2010 Worksheet 12

1. Consider the following graphs:

Which graphs contain an Eulerian cycle? What about a Hamiltonian path?

2. In the graph from problem 1, find the size of the maximum independent set and the size of the largest clique.

3. Prove or disprove: Every disconnected graph has an isolated vertex.

4. Under what conditions does the graph $K_{m,n}$ - the complete bipartite graph on m + n vertices, where the partite sets have m and n vertices and every possible edge is drawn in - have an Eulerian circuit? (In other words, what must be true about m and n?)

5. Show that in a simple graph with at least 2 vertices, there must be two vertices that have the same degree.

6. Suppose that v is an endpoint of a cut edge. Prove that v is a cut vertex if and only if d(v) > 1.