Math 135 - Sets (part 2)

Note Title 9/13/2010

Announcements

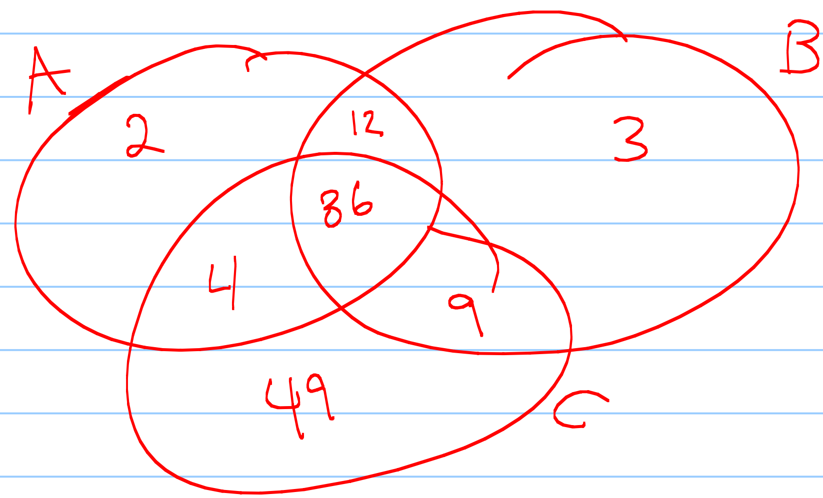
- HW 2 due today
- HW 3 will be posted this afternoon
  due next ~~Wednesday~~ Friday

Venn Diagrams

these are a picture, not a proof

Sometimes we want a picture of how sets interact.

Ex: $A = \{n \in \mathbb{N} : n \text{ is even}\}$
$B = \{n \in \mathbb{N} : n \text{ is divisible by } 3\}$
$C = \{n^2 : n \in \mathbb{N}\}$

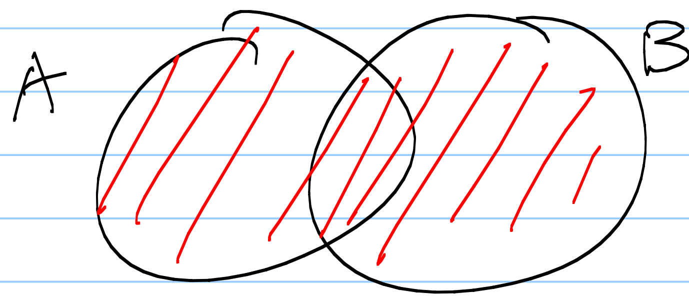


$4 \in A$
$4 \in C$

More Definitions

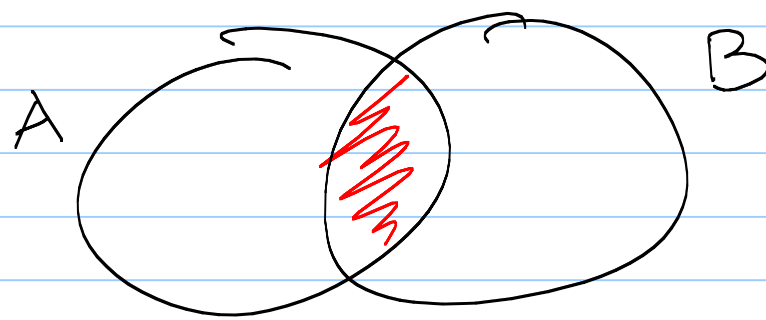
$A \cup B = B \cup A$

<u>Union</u>: $A \cup B = \{x \mid x \in A \vee x \in B\}$

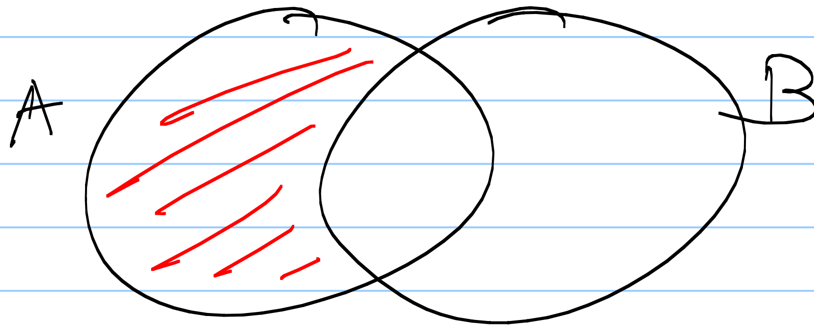


$A \cap B = B \cap A$

<u>Intersection</u>: $A \cap B = \{x \mid x \in A \wedge x \in B\}$



Set Difference: $A - B = \{x \mid x \in A \overset{\wedge}{\text{and}} x \notin B\}$



Dfn: Two sets are called **disjoint** if their intersection is empty, i.e. $A \cap B = \emptyset$.

(or $A - B = A$)

Examples

$$A = \{2, 7, \{a,b\}, \pi\}$$
$$B = \{\sqrt{2}, \pi, a, b\}$$
$$C = \{\{a\}, b, \{a,b\}\}$$

$$A \cup B = \{2, 7, \{a,b\}, \pi, \sqrt{2}, a, b\}$$

$$A \cap B = \{\pi\}$$

$$\overbrace{(A \cap C)}^{\{\{a,b\}\}} \cup B = \{\sqrt{2}, \pi, a, b, \{a,b\}\}$$

$$B - C = \{\sqrt{2}, \pi, a\}$$

$$2^{A \cap B} = \{\emptyset, \{\pi\}\}$$

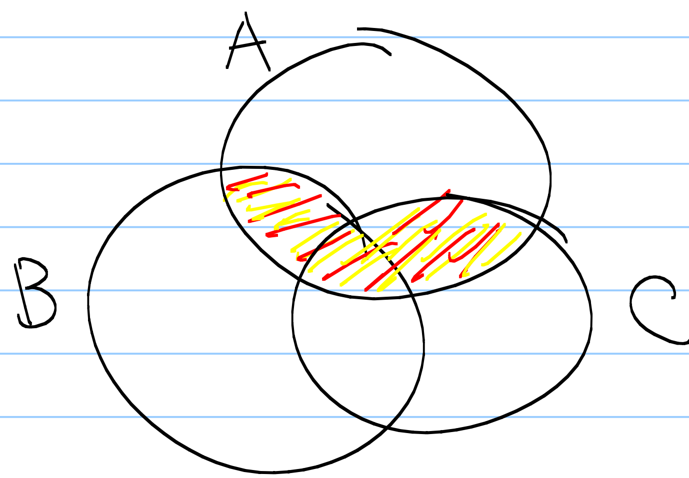
Set identities (p. 124)

~~De Morgan's Law~~

Thm: For all sets A, B & C,

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

(so $\cap$ distributes over $\cup$)



← Not a proof

Proof: Show (1) $A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$

and (2) $(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$

(1) Take $x \in A \cap (B \cup C)$

Need to show it is also $\in (A \cap B) \cup (A \cap C)$

We know $x \in A$ and $x \in (B \cup C)$

$x \in B$ or $x \in C$

$x \in A$ and $(x \in B$ or $x \in C)$

So either $x \in A$ and $x \in B$

or $x \in A$ and $x \in C$

$\Rightarrow x \in (A \cap B) \cup (A \cap C)$

② $(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$

Take $x \in (A \cap B) \cup (A \cap C)$

x is in A and B
or x is in A and C

Either way, $x \in A$

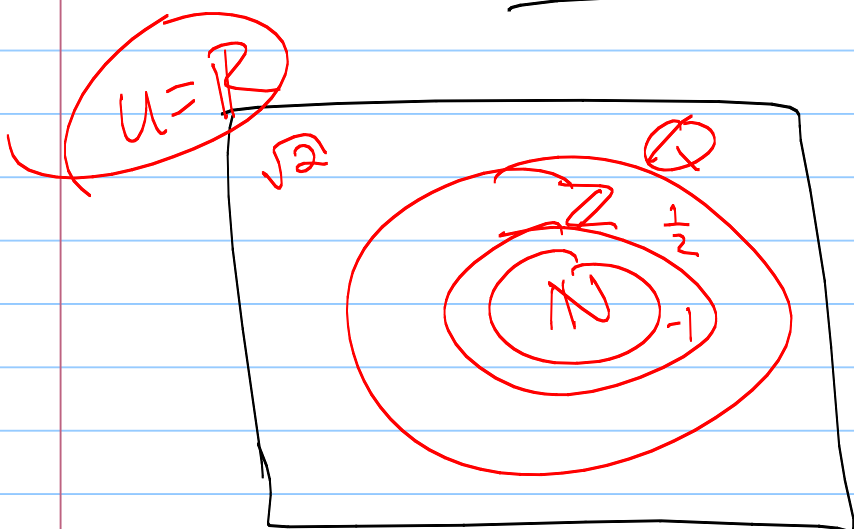
Also, x is either in B or in C.

$\Rightarrow x \in A \cap (B \cup C)$

$\blacksquare$

The Universe

Many times, all of the sets we are interested in come from a single large set called the <u>universe</u>.



$U = \mathbb{R}$

$\mathbb{Z}$

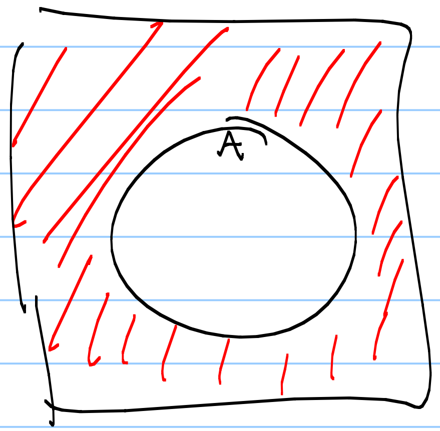
$\mathbb{Q}$

$\mathbb{N}$

Complementation: Relative to U, the complement of A is

$$\bar{A} = U - A = \underbrace{\{x \in U : x \notin A\}}$$

U



Ex: $\mathbb{Z} - \mathbb{N}$ = negative integers

$\mathbb{R} - \mathbb{N}$

De Morgan's Laws

☆ - $\overline{A \cup B} = \overline{A} \cap \overline{B}$

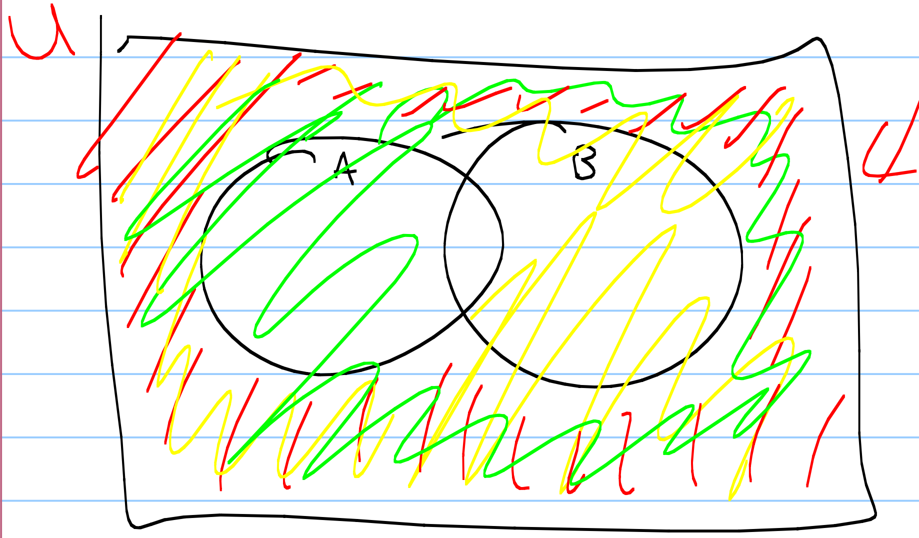
☆ - $\overline{A \cap B} = \overline{A} \cup \overline{B}$

prove this one

← (look familiar?)

$\neg(p \wedge q) = \neg p \vee \neg q$

$\neg(p \vee q) = \neg p \wedge \neg q$



$x \notin A$

Prove that $\overline{A \cap B} = \overline{A} \cup \overline{B}$.

pf: How do we show two sets are equal?

① $\overline{A \cap B} \subseteq \overline{A} \cup \overline{B}$

Take $x \in \overline{A \cap B}$, so $x \in U$ and $x \notin A \cap B$,

x is not in A and B

so either x is not in A
or x is not B

$x \in \overline{A}$ or $x \in \overline{B}$

$\Rightarrow x \in \overline{A} \cup \overline{B}$

$x \notin A \stackrel{?}{\Longleftrightarrow} x \in \overline{A}$ by definition

② $\overline{A} \cup \overline{B} \subseteq \overline{A \cap B}$

Take $x \in \overline{A} \cup \overline{B}$

$\Rightarrow x \notin A$ or $x \notin B$

Logic version of D.M.L.

$(\neg p \vee \neg q) \equiv \neg (p \wedge q)$

$\Rightarrow x \notin (A \text{ and } B)$

$\Rightarrow x \in \overline{A \cap B}$

Notation:

$B \cup A = A \cup B$

We will write

$$\bigcup_{i=1}^{n} A_i = ((A_1 \cup A_2) \cup \cdots \cup A_n)$$

Similarly,

$$\bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap \cdots \cap A_n$$