Math 135 - Sets

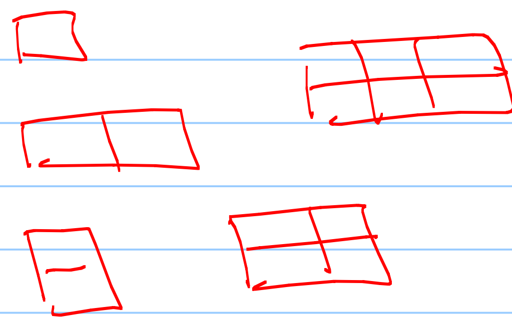
Note Title

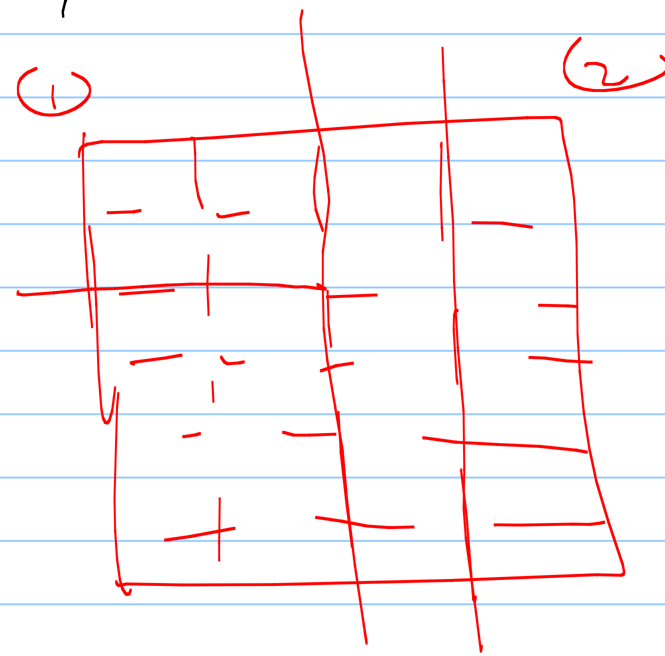
9/10/2010

Announcements

- HW due Wednesday

1-2 Office Hours





Sets (2.1)

<u>Dfn</u>: A <u>set</u> is an (unordered) collection of objects.

<u>Ex</u>: $\phi = \{\}$ (the empty set)

$\{1, 3, 5, 7\}$

$\{1, 2, 3, \ldots, 1000\}$

$\{a, b, c, \ldots, z\}$

$\{\phi, \{1\}, \{1, 3\}, \{1, 3, 5\}\}$

Definitions

- A set is said to contain its elements (or members).
- Two sets are equal if & only if they contain the same elements.

Ex: $\{1,3,5,7\} = \{3,7,5,1\}$

$\{1,3,5,7\} = \{1,1,3,3,5,7\}$

(order & multiplicity of members don't matter!)

Examples

Natural Numbers: $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$

Integers: $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

Rational Numbers: $\mathbb{Q} = \{p/q : p \in \mathbb{Z}, q \in \mathbb{Z}, \text{ and } q \neq 0\}$

Real Numbers: $\mathbb{R}$ (includes $\mathbb{Q}$ plus π, $\sqrt{2}$, etc...)

Ways to define a set

- List: $S = \{1, 5, 11\}$

  $T = \{1, 2, 3, 4, \ldots\}$

- English: "Let S be the set of squares."
- Form description: $S = \{n^2 : n \in \mathbb{N}\}$

  " : " such that

- Property description: $S = \{n \in \mathbb{N} : n \text{ is a perfect square}\}$

$S = \{0, 1, 4, 9, 16, 25 \ldots\}$

Notation:

member of

- $x \in S$ means x is a member of S
- $x \notin S$ means x is not a member of S
- $A \subseteq B$ means that A is a subset of B

↳ That is, $\forall x, (x \in A \rightarrow x \in B)$

Note: $A = B \longleftrightarrow (A \subseteq B$ and $B \subseteq A)$

- $A \subsetneq B$ or $A \subset B$ means A is a proper subset of B

(so $A \subseteq B$ and $A \neq B$)

Examples:

$\mathbb{Z} \subseteq \mathbb{Z}$

$\mathbb{N} \subseteq \mathbb{Z}$ $\leftarrow$ $\mathbb{N} \subset \mathbb{Z}$

$\sqrt{5} \in \mathbb{R}$

$\sqrt{2} \notin \mathbb{Q}$ $\leftarrow$ we proved this!

Lemma: For any set S, $\emptyset \subseteq S$.

proof: Show: For all x, if $x \in \emptyset$ then $x \in S$.

$\emptyset$ has no elements!

So $\emptyset \subseteq S$. $\blacksquare$

Note: I am <u>not</u> saying $\cancel{\emptyset \in S}$.

$\{1, 2\} \subseteq \{1, 2, 3\}$

<u>not</u> $\cancel{\{1, 2\} \in \{1, 2, 3\}}$

$\{1, 2\} \in \{\emptyset, \{1, 2\}\}$

Sets: some more definitions

Let S be a set. If S has exactly n (unique) elements, then we say S is finite, with cardinality n, written $|S| = n$.

S is said to be infinite if it is not finite.

What are infinite sets?

$\mathbb{Z}, \mathbb{N}, \mathbb{R}, \mathbb{Q}$

Dfns (cont)

The power set of S, $P(S)$ or 2^S, is the set of all subsets of S.

Ex: Let $S = \{0, 1, 2\}$. What is the power set of S?

$$2^S = \{ \{0\}, \{1\}, \{0,1\}, \{1,2\}, \{0,2\}, \{0,1,2\}, \{2\}, \phi \}$$

Ex: Let $S = \{a, 1, \sqrt{2}\}$. What is 2^S?

$$2^S = \{\emptyset, \{a\}, \{1\}, \{\sqrt{2}\}, \{a, 1, \sqrt{2}\}, \{a, 1\}, \{a, \sqrt{2}\}, \{1, \sqrt{2}\}\}$$

Ex: What is the power set of $\emptyset$?

$$2^{\emptyset} = \{\{\}\} = \{\emptyset\} \neq \emptyset$$

$$S = \{\phi, \{1,2\}, \sqrt{2}\}$$

$$2^S = \{\phi, \{\phi\}, \{\{1,2\}\}, \{\sqrt{2}\}, \{\phi, \{1,2\}\}, \{\phi, \sqrt{2}\}, \{\{1,2\}, \sqrt{2}\}, \{\phi, \{1,2\}, \sqrt{2}\}\}$$