Math 135 - More recurrences

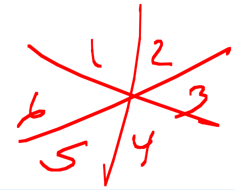
Note Title 10/25/2010

Announcements

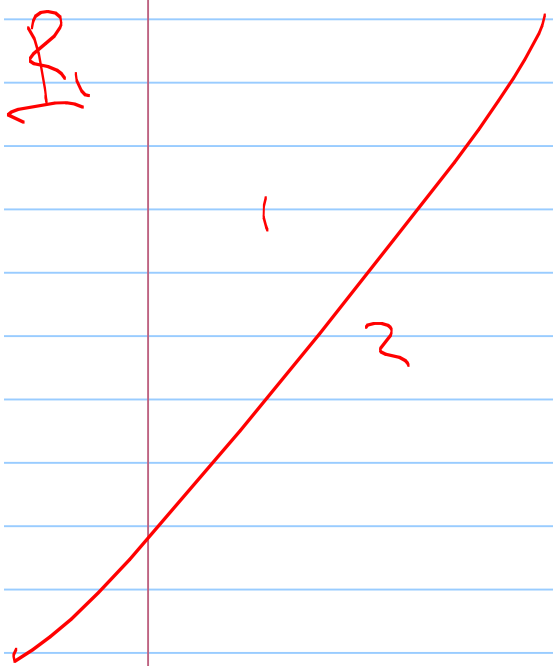
- HW due Friday

Counting regions in the plane:

(Assume no 3 lines meet at a single point)



n=1 lines	n=2 lines	n=3 lines
R_1	R_2	R_3



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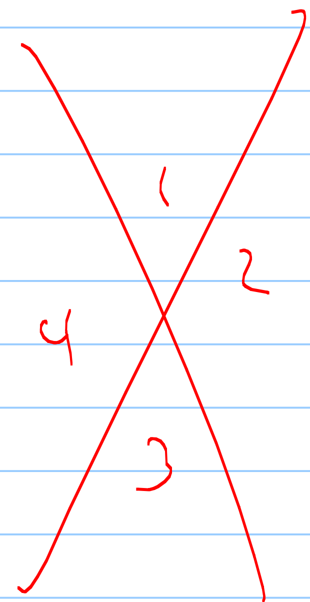

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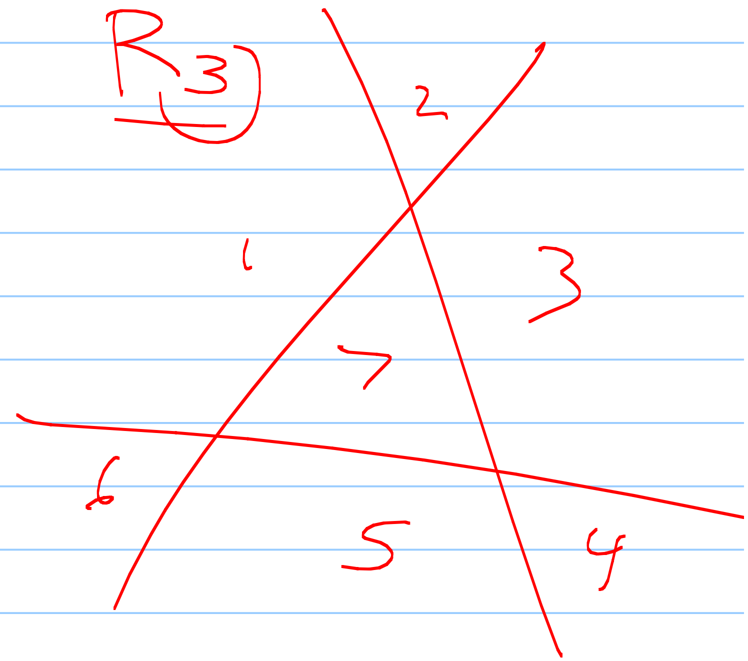

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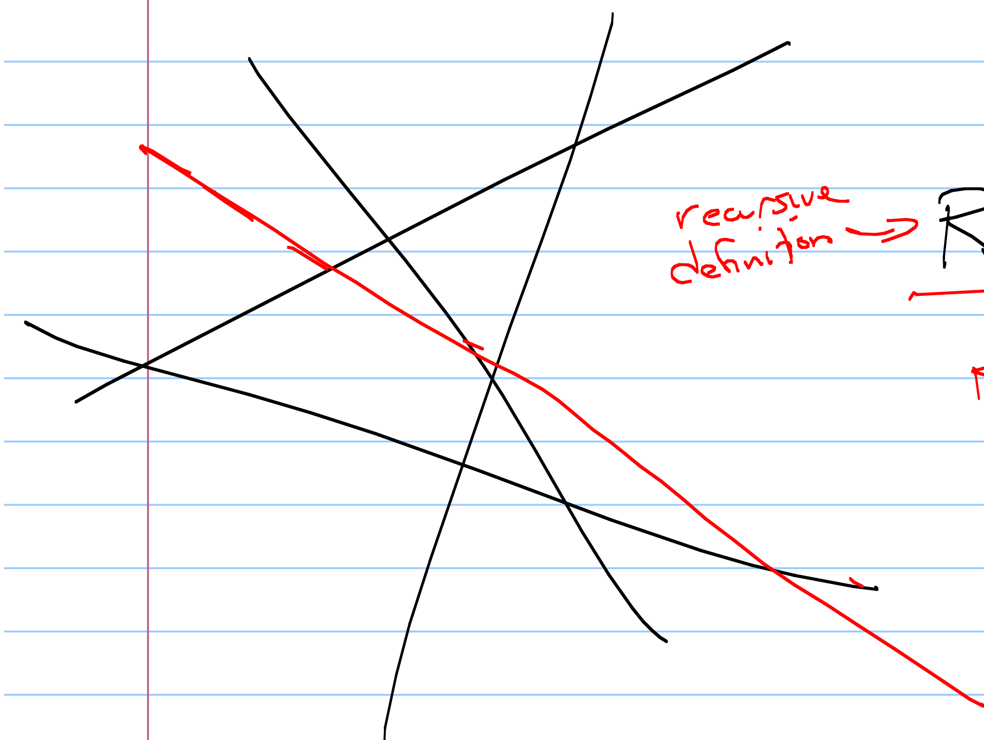
Consider $n-1$ lines w/ R_{n-1} regions.
What happens when we add an n^{th} line?
(Assume no parallel lines, so every line crosses every other line)



recursive definition $\rightarrow$ $R_n = \underbrace{R_{n-1}} + n$

$R_3 = R_2 + 3$

new line intersects $n-1$ other lines
divides a region into 2 regions between intersections

$R_k = R_{k-1} + k$ $\qquad$ $\underline{R_1 = 2}$

<u>Unrolling</u>: $R_n = R_{n-1} + n$

$= R_{n-2} + (n-1) + n$

$= R_{n-3} + (n-2) + (n-1) + n$

$= R_{n-4} + (n-3) + (n-2) + (n-1) + n$

$\vdots$

$= R_i + (i+1) + (i+2) + \cdots + n$

$\vdots$

$= R_1 + 2 + 3 + 4 + \cdots + n$

$= 2 + 2 + 3 + 4 + \cdots + n$

$$R_1 = 2$$
$$R_k = R_{k-1} + k$$

<u>Claim</u>: $R_n = 1 + \frac{n(n+1)}{2}$

<u>Pf</u>: induction on n

<u>Base Case</u>: $R_1 = 1 + \frac{1(2)}{2} = 2$ ✓

<u>IH</u>: Assume $R_{n-1} = 1 + \frac{(n-1)(n)}{2}$

<u>IS</u>: $R_n = R_{n-1} + n$

use IH $= \left(1 + \frac{(n-1)n}{2}\right) + n$

$$= 1 + \frac{(n-1)(n) + 2n}{2} = 1 + \frac{n(n-1+2)}{2}$$

$$= 1 + \frac{n(n+1)}{2}$$

$\boxed{}$

Fibonacci numbers:

Definition $\begin{cases} F_0 = 0 \\ F_1 = 1 \\ F_k = F_{k-1} + F_{k-2} \end{cases}$

Identity: $\underbrace{\sum_{k=1}^{n} F_k}$ $=$ $\underbrace{F_{n+2} - 1}$

prove using induction on n:

Base case: $n = 1$

LHS: $\sum_{k=1}^{1} F_k = F_1 = 1$

$F_3 - 1 = 2 - 1 = 1$ ✓

IH: Assume $\sum_{k=1}^{n-1} F_k = F_{(n-1)+2} - 1$

$= F_{n+1} - 1$

IS:

$$\sum_{k=1}^{n} F_k = \underbrace{\sum_{k=1}^{n-1} F_k}_{IH} + F_n$$

$$= F_{n+1} - 1 + F_n$$

$$= F_{n+2} - 1$$

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