Math 135 - Discrete Math

Note Title 1/11/2010

- Syllabus is posted
- HW0 - due <u>this</u> <u>Friday</u>

<u>Logic</u>

<u>Dfn</u>: A <u>proposition</u> is a declarative statement which is either true or false, but not both.

<u>Ex</u>: The sky is blue.

Money is worthless.

Ambiguous: I like ice cream.

The sky is pretty.

Negation $\neg p$

Let p be a proposition.
The **negation** of p, written $\neg p$, is the statement:

"It is not the case that p"

Ex: p = "The sky is yellow"

$\neg p$ = "The sky is not yellow"

Conjunction and Disjunction
(ie "and" and "or")

AND <u>Conjunction</u>: "p and q", written $p \wedge q$, is true exactly when <u>both</u> p & q are true & is false otherwise

OR <u>Disjunction</u>: "p or q", written $p \vee q$, is true if either p or q is true, or if both are true

Truth Tables

or

and

p	q	$p \vee q$	$p \wedge q$
T	T	T	T
T	F	T	F
F	T	T	F
F	F	F	F

Exclusive Or: $p \oplus q$

True when exactly one of p & q is true (but not both).

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Implications: $p \to q$

"if p, then q"

"p implies q"

"q if p"

Ex: "If I am elected, then I will lower taxes."

When is it true?

<u>Dfn:</u> $p \to q$ is false when p is <u>true</u> and q is <u>false</u>

(and is true otherwise!)

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

vacuously true

Examples to ponder:

"If it is sunny, then we will go to the beach."

"If today is Friday, then $2+5=7$."

Converse, Inverse, + Contrapositive

The <u>converse</u> of $p \to q$ is the statement $q \to p$.

The <u>inverse</u> is $\neg p \to \neg q$. ←

The <u>contrapositive</u> is $\neg q \to \neg p$.

<u>Exercise</u>: Draw the truth tables.

p	q	$p \to q$	$q \to p$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

Converse

Inverse

Contrapositive

$\neg p$	$\neg q$	$\neg p \to \neg q$	$\neg q \to \neg p$
F	F	T	T
F	T	T	F
T	F	F	T
T	T	T	T

Logical Equivalence

Propositions that have the same truth values are called **logically equivalent**. (written $p \equiv q$).

Example?

A statement is equivalent to its contrapositive.

Example: $\neg(p \vee q) \equiv \neg p \wedge \neg q$

Why? DeMorgan's Law

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T