Math 135 - Graphs (part 3)

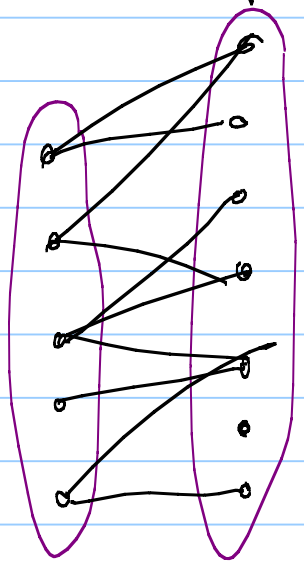
Note Title 4/21/2010

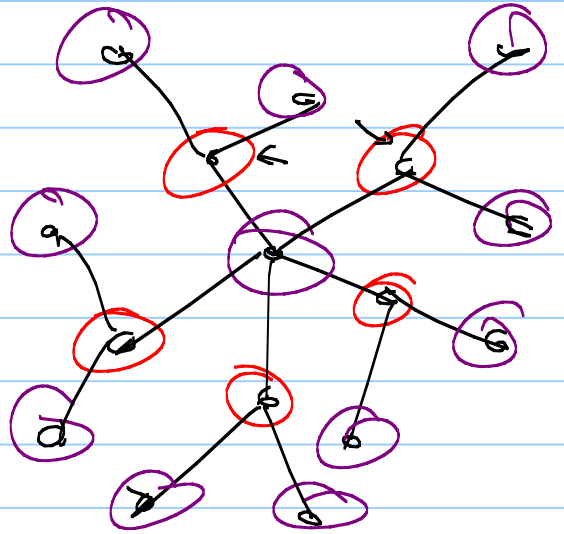
Announcements

- Have last HW due Monday
- Review session Tuesday 1-2 (check website)

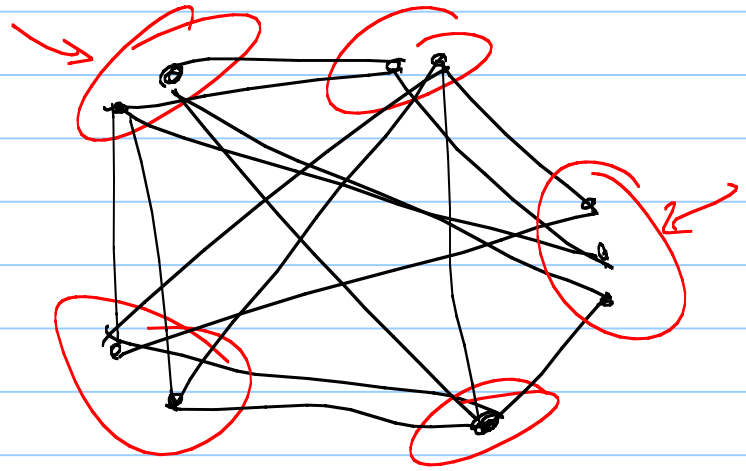
Def: A graph G is **bipartite** if the vertices in G can be partitioned into 2 independent sets.

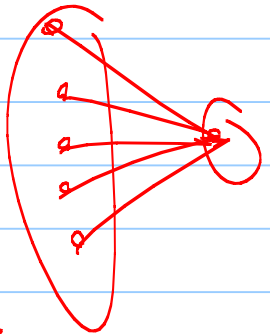
Ex:





Dfn: A graph is k-partite if its vertices can be partitioned into k independent sets.



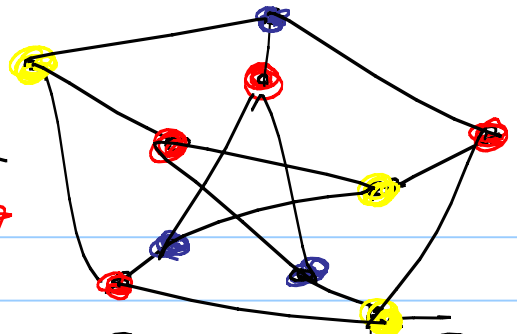


5-partite

(Any graph is n-partite)

$\chi(G) = 3$?

3-colorable



Colorability

A graph is k-colorable if we can color each vertex with one of k colors so that adjacent vertices get different colors.

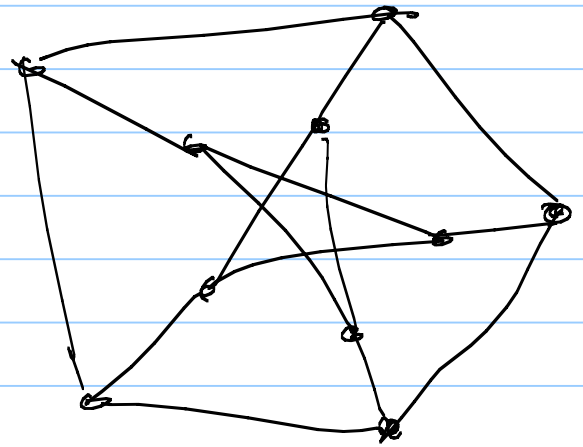
Thm: G is k-partite $\Leftrightarrow$ G is k-colorable.

pf:

Dfn: The chromatic number of a graph is the minimum k s.t. G can be k-colored.

(Written $\chi(G)$.)

chi (G)



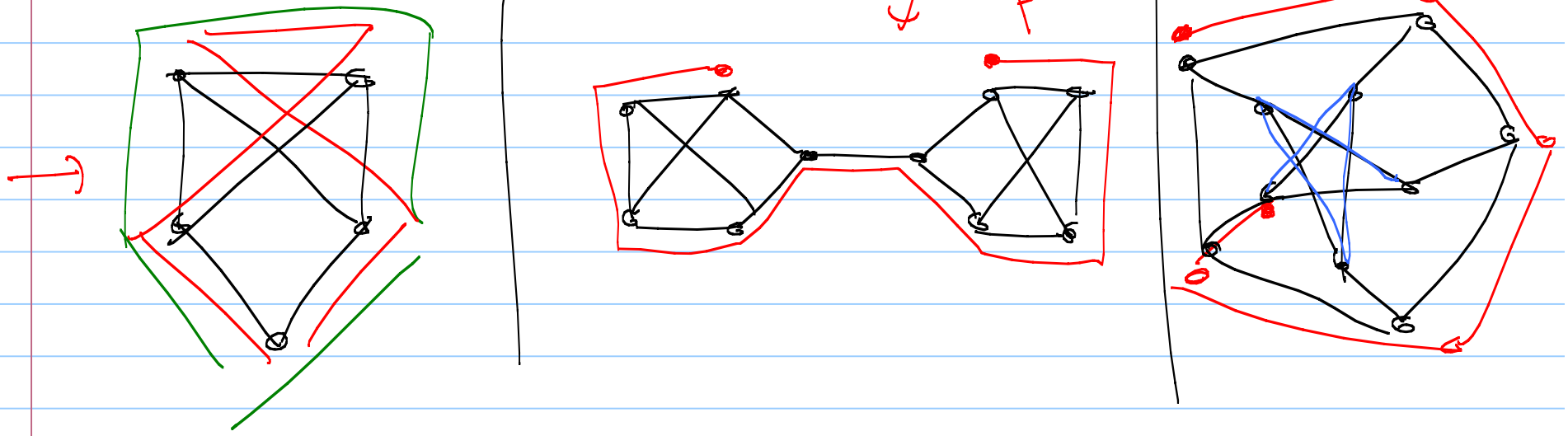
Cor: G is bipartite $\iff \chi(G) \leq 2$.

Why? from prev. thm

Thm: G contains an odd cycle $\iff$ G is not 2 colorable (ie not bipartite).

Hamiltonian Graphs

Dfn: A **Hamiltonian cycle** is a cycle that visits every vertex exactly once.
Graphs with such a cycle are called **Hamiltonian**.



Surprisingly, no known (fast) way to check for these!

The best known algorithms are essentially brute force – try every possible ordering.

(This is an NP-Complete problem.)