Math 135 - Graphs (part 2)

Note Title 4/21/2010

Announcements

- Final exam is next Wed., noon - 2pm
- Review session on Tuesday 1-2pm
- HW due Monday

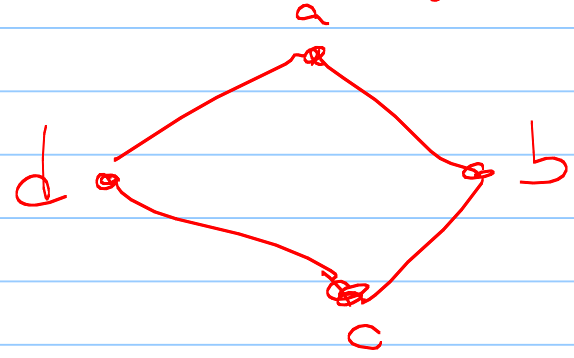
Last time: $G = (V, E)$

vertex set (V), edge set (E)

$V = \{a, b, c, d\}$

E is a set of subsets

$E = \{\{a b\}, \{b c\}, \{c d\}, \{a d\}\}$



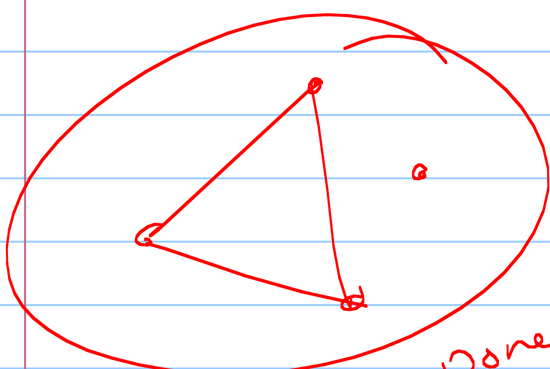
Dfn: A graph G is connected if for every pair of vertices u + v, there is a u-v walk in G.

$\to$ sequence of edges

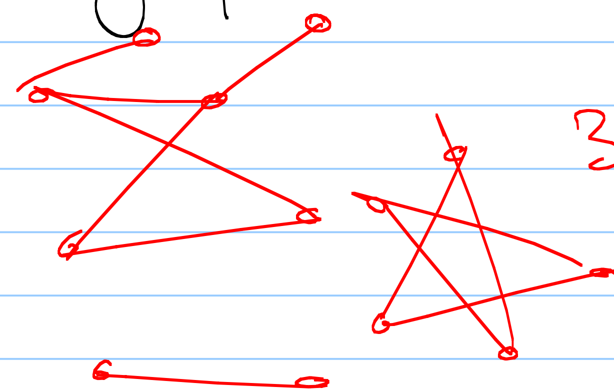
$\{v_1, v_2\} - \{v_2, v_3\} - \{v_3, v_4\}$

$\{v_i, v_{i+1}\}$

The components of G are maximally connected subgraphs.



2 components



3 components

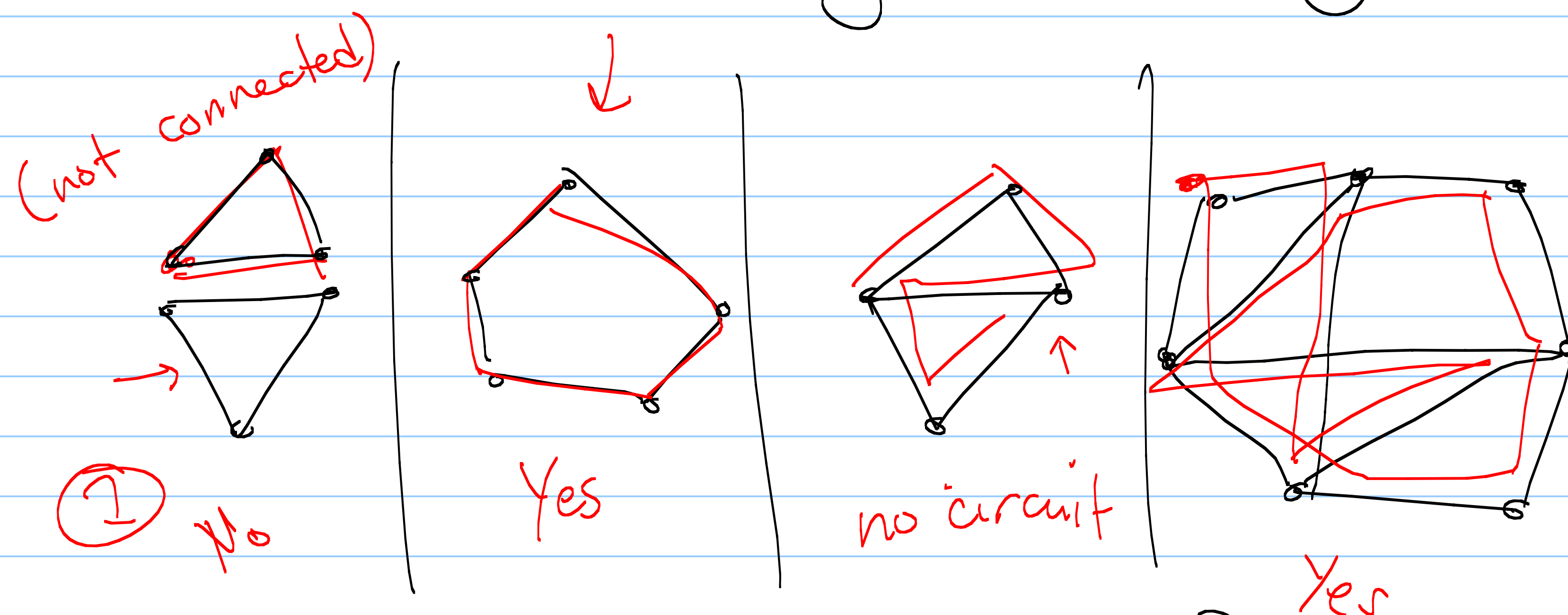
walk - set of edges where we "walk" around
allowed to repeat edges & vertices

path - walk with no repeated vertices
or edges

cycle - a path that ends at same
vertex it began at

circuit - walk that ends where it began
(allowed to repeat edges & vertices)

Dfn: An Eulerian circuit is a circuit which uses every edge exactly once.



What graphs have these?

Thm: A graph G has an Eulerian circuit if & only if G is connected & every vertex has even degree.

pf: **⇒):** Suppose G has Eulerian circuit.

- know it is connected, since circuit gives a walk between any 2 vertices
- odd degree vertex would circuit would come in & not beable to leave, since every time vertex appers on circuit, it needs 2 edges.

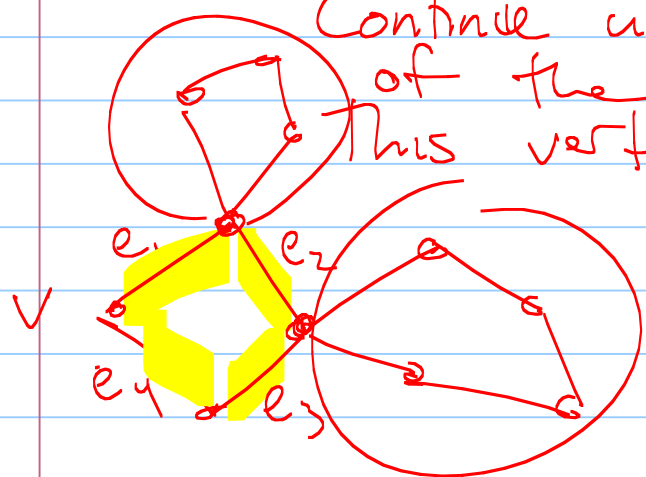
$\Leftarrow$: Know G is connected, + every vertex has even degree.

Start at a vertex v.

Follow an edge. New vertex has even degree, so follow another edge.

Continue until no unused edges out of the current vertex.

This vertex must be v.



Delete edges I have traveled so far.

Rest not necessarily connected

Rest still has even degree.

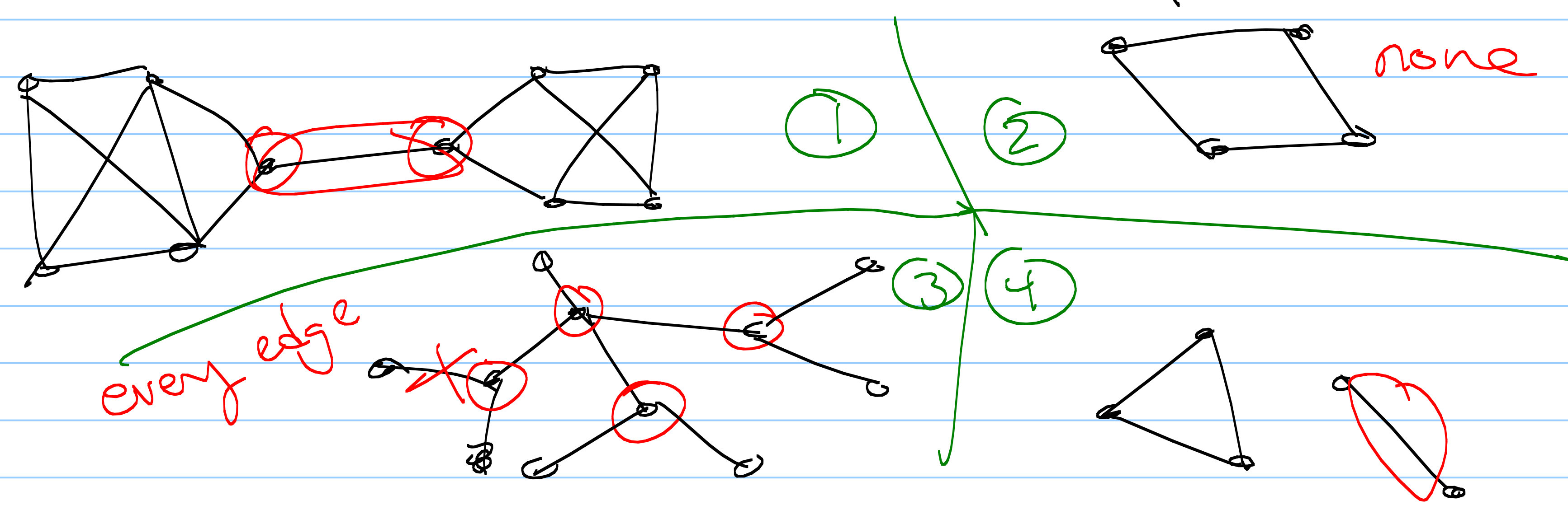
So continue on those components.

At end, can connect into an Euler circuit.

∎

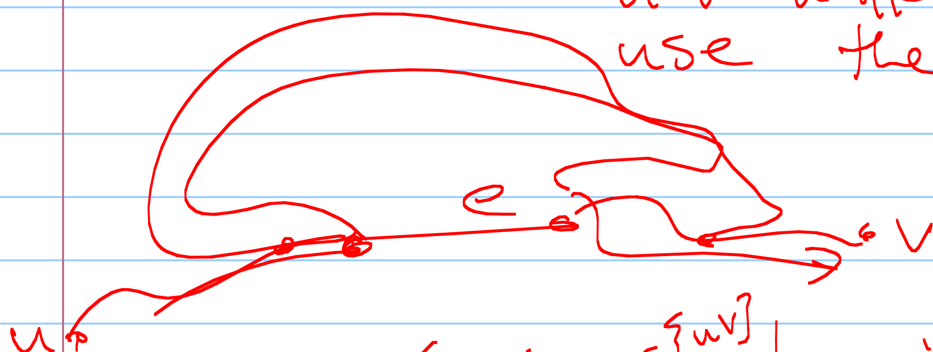
Dfn: A cut-edge in a graph is an edge whose deletion increases the number of components.

A cut-vertex is a vertex whose deletion increases the # of components.

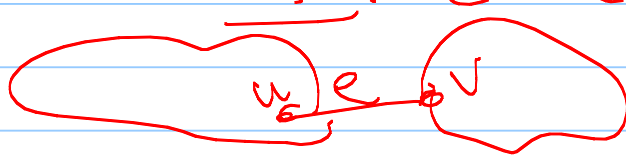


Thm: An edge is a cut edge $\iff$ it does not belong to any cycle.

pf: $\Rightarrow$ Suppose e is a cut edge.
if e is on a cycle, any u-v walk that used e can use the rest of cycle



$\Leftarrow$: $e=\{u,v\}$ doesn't belong to any cycle.



if u + v have a walk between them, then e is on a cycle $\lightning$

max # of edges in a graph with n vertices is $\binom{n}{2} = \frac{n(n-1)}{2}$

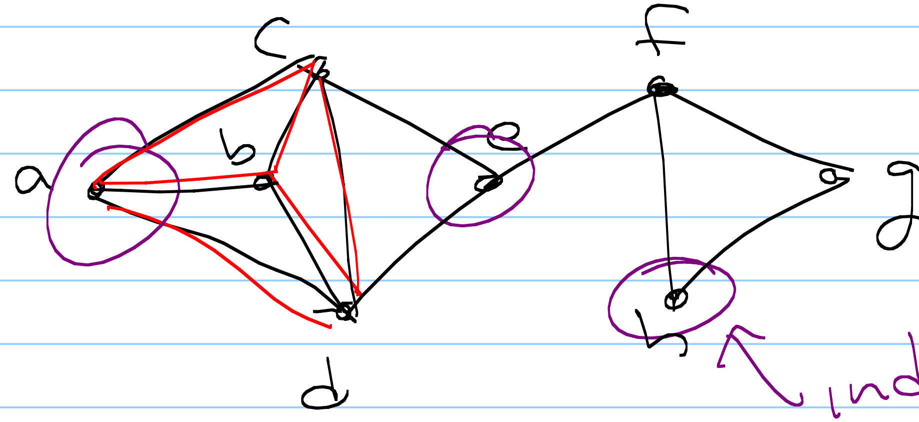
Dfn: In a graph G, a clique is a set of vertices that are pairwise adjacent.

An independent set is a set of vertices that are pairwise non-adjacent.

clique of size k

how many edges?

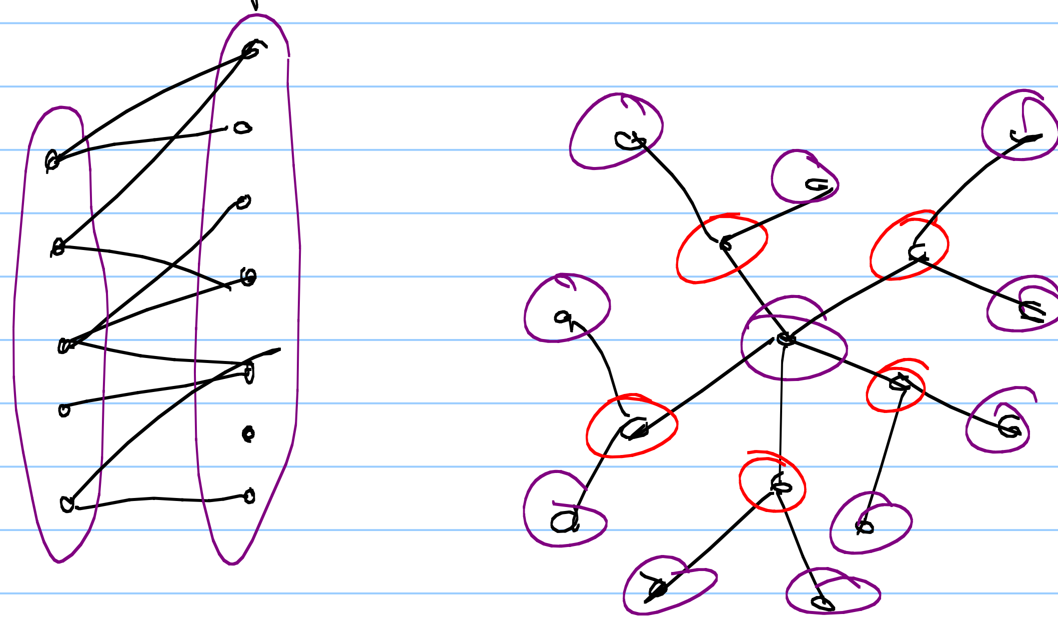
$\binom{k}{2}$



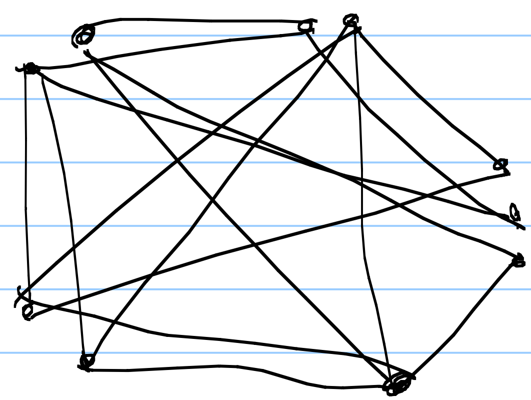
ind. set of size 3

<u>Dfn</u>: A graph G is **bipartite** if the vertices in G can be partitioned into 2 independent sets.

<u>Ex</u>:



Dfn: A graph is <u>k-partite</u> if its vertices can be partitioned into k independent sets.



Colorability

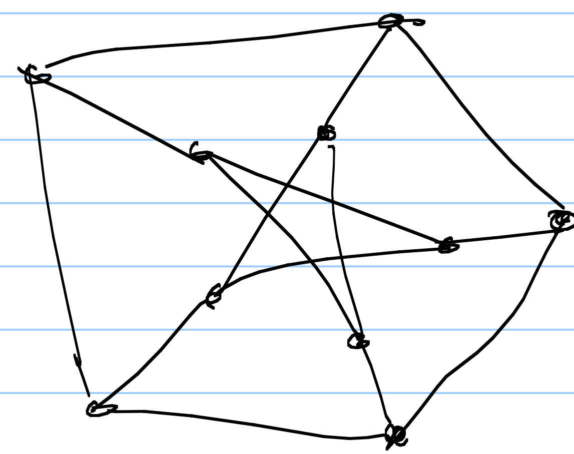
A graph is k-colorable if we can color each vertex with one of k colors so that adjacent vertices get different colors.

Thm: G is k-partite $\Leftrightarrow$ G is k-colorable.

pf:

Dfn: The <u>chromatic number</u> of a graph is the minimum k s.t. G can be k-colored.

(Written $\chi(G)$.)



Cor: G is bipartite $\iff \chi(G) \le 2$.

Why?