Math 135 - Graphs

Note Title 4/14/2010

Announcements

- HW due tomorrow
  drop by my office before 5pm
- 1 more HW due next Monday
- Final next Wed at 12-2
  8th
- Review on Tuesday at 1pm
  location TBA
- Today: graded midterms
  average: 44.4/60 or ~ 74%

Worksheet Recap:

(1b) How many ~~palindromes~~ bitstrings palindromes of length n are there?

(assume n is even)

$2^{n/2}$: $\underbrace{\underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}}_{n/2 \text{ choices}}\ \underline{x}\ \underline{x}\ \underline{x}\ \underline{x}\ \underline{x}$

0 / 1

(1d) A group contains n men & n women. How many ways are there to arrange them in a row if they must alternate?

$\underline{n} \cdot \underline{n} \cdot \underline{n-1} \cdot \underline{n-1}$ __ __ __ __ __ __

man or woman

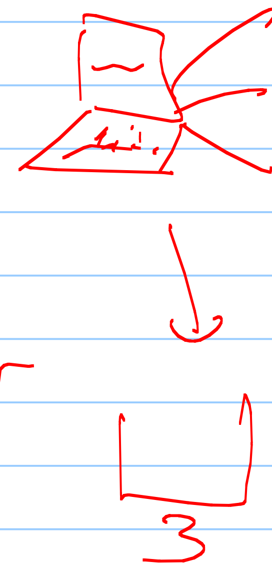
2n spots

n! ways to arrange women

n! ways to arrange men

$= 2 \cdot n! \cdot n!$

② A network has 6 computers, each connected to 0 or more other computers. Show that at least 2 in the network connected to the same # of other computers.

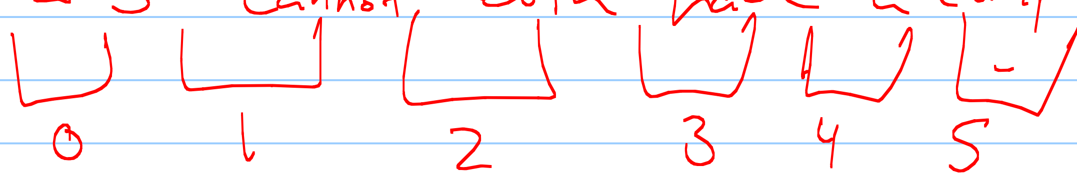


pigeonhole

6 ← boxes: # of connections

6 ← balls: computers

only 5 boxes can be occupied, since 0 & 5 cannot both have a computer



(3) 100 tickets sold to 100 different people.
4 prizes (grand prize is trip to Hawaii)
How many ways to award if:

$\binom{100}{4}4!$ — no restrictions? $100 \cdot 99 \cdot 98 \cdot 97 = P(100, 4)$

$\binom{100}{3}3!$ — person holding 47 wins the grand prize? $99 \cdot 98 \cdot 97 = P(99, 3)$

$\binom{99}{4}4!$ — person with 47 doesn't win? $P(99, 4)$

$\binom{4}{1}\binom{99}{3}3!$ — person with 47 wins some prize? $4 \cdot 99 \cdot 98 \cdot 97 = 4 \cdot P(99, 3)$

— both 19 + 47 win a prize? $4 \cdot 3 \cdot 98 \cdot 97$

4

$$\underbrace{\binom{n}{k}\underbrace{k}}_{\text{choosing subcommittee of k people from n people, then electing a chair}} = \overbrace{n\binom{n-1}{k-1}}^{\text{choose a chair then choose the rest of the committe}}$$

Graphs Ch 9

Motivation: Model relationships or connections

- Cities & roads
- Internet Connectivity
  (routers, computers, etc.)
- Webpage links
- Social Networks
- Biological Networks

  ⋮

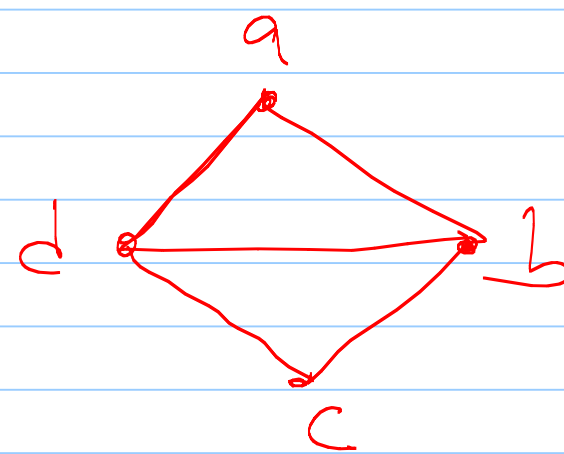
Dfn: A graph $G=(V,E)$ is a pair of sets:

- V is a set of vertices
- E is a set of edges

Each edge is associated with 2 vertices, called its endpoints.

Ex: $V=\{a,b,c,d\}$

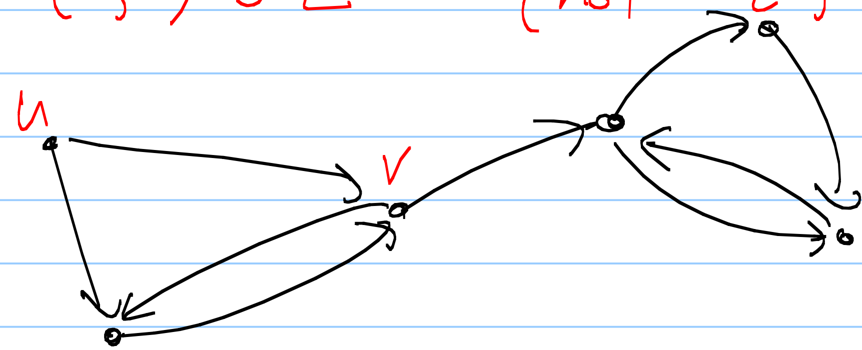
$E=\{\{ab\},\{bc\},\{cd\},$
$\{ad\}, \{bd\}\}$



In a directed graph, each edge is an ordered pair - not just a set.

Ex:

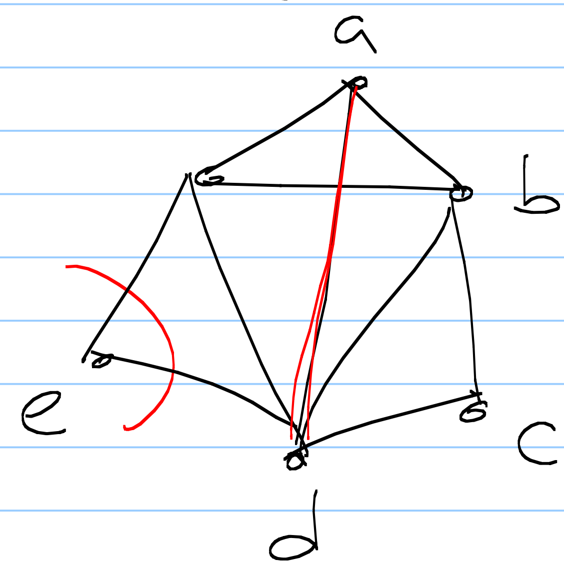
$(u,v) \in E$ (not $\{u,v\} \in E$)



one way streets

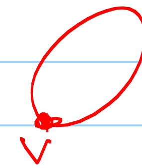
flight lanes

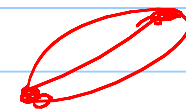
Dfn: We say an edge is incident to its endpoints, & two vertices are adjacent if there is an edge between them.



a & d are adjacent

e has 2 incident edges

We can have **loops**:  $\{v, v\} \in E$

or multiple **edges**: 

A graph is called **simple** if it has no loops or multiple edges.

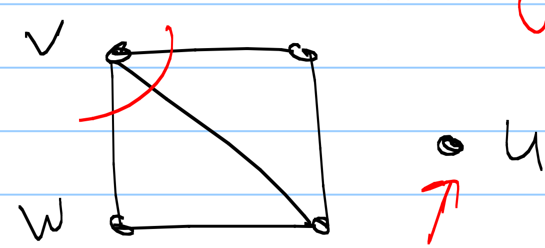
We'll (usually) deal with simple, undirected graphs here.

not (u,v) but $\{u,v\}$

Dfn: The degree of a vertex, $d(v)$, is the number of incident edges.

5 edges

$d(v) = 3$



$d(u) = 0$

$d(\text{vertex})$ is between 0 & # vertices -1

$G=(V,E)$

Thm: $$\sum_{v\in V} d(v) = 2|E|$$ ← # of vertex-edge incidences

Degree Sum formula, or Handshaking Lemma

pf: combinatorial proof:

$\sum_{v\in V} d(v)$ — counting # of edges incident to each vertex

$2|E|$ — each edge has 2 incident vertices, so

$$\underbrace{2+2+2+\cdots+2}_{|E|} = 2|E|$$

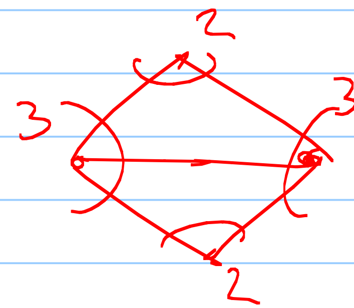
Thm: In a simple, undirected graph, the number of nodes with odd degree is even.

pf: Know:

even + even = even

even + odd = odd

odd + odd = even



Know $d(v_1) + d(v_2) + \cdots + d(v_n)$ = even #