

Math 135 - Induction

Note Title

9/9/2010

Announcements

- HW2 due ~~Monday~~ Wednesday
- Next HW out Monday or Tuesday
(due ~ 1 week after)
- First midterm in ~2 weeks (?)
or 3

Recap: Induction

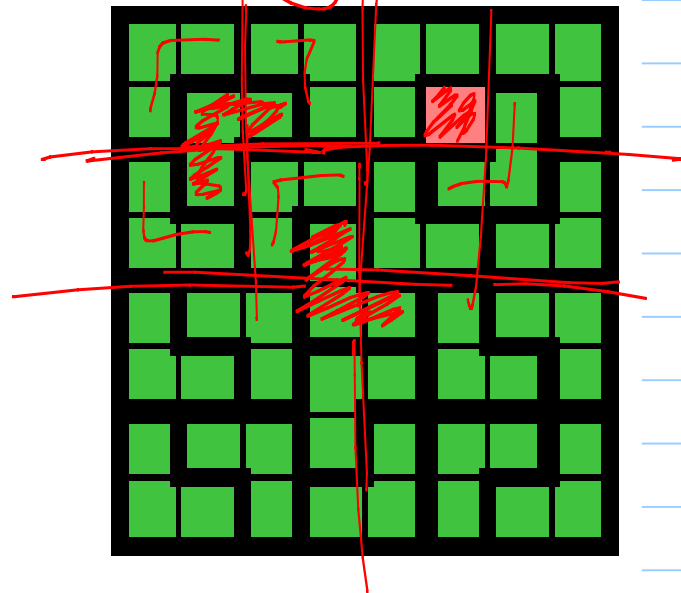
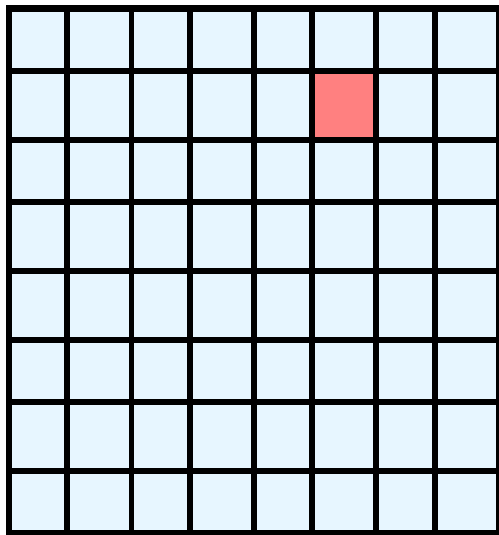
① Base Case

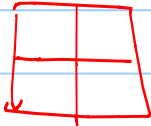
② I H

③ I S

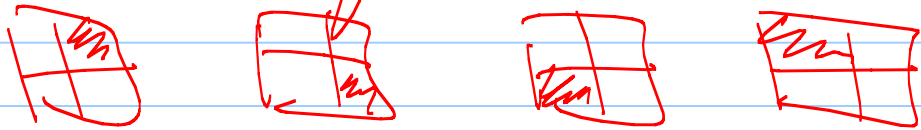
Let n be a positive integer. Show that any $2^n \times 2^n$ chess board with one square removed can be tiled with L shaped pieces.

Show using induction on n .



Base case: $n=1$, so 

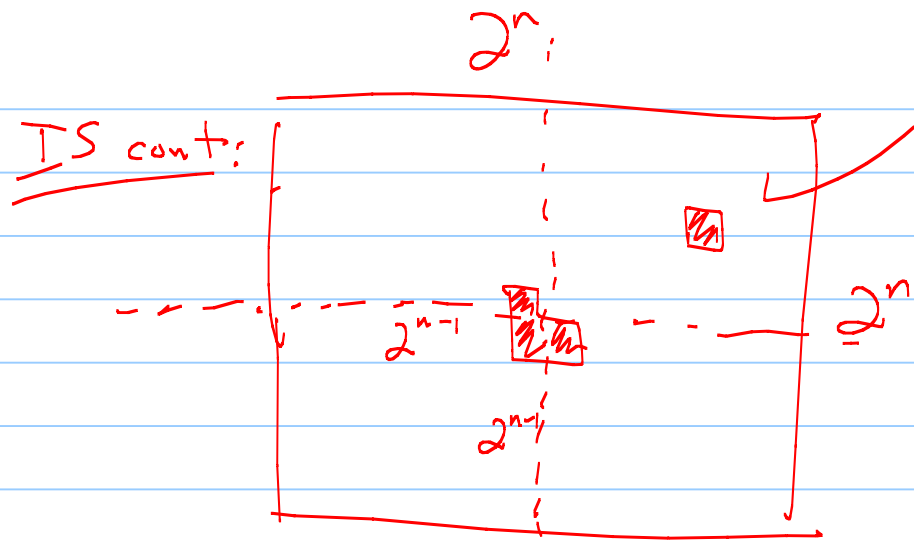
✓ 4 possible squares to remove



each is covered by one L-piece.

✓ IH: Any 2^{n-1} by 2^{n-1} chessboard with 1 piece removed can be tiled by L-shaped pieces.

IS: Consider $2^n \times 2^n$ board with 1 piece removed.



Divide into 4 quadrants.
 The one with a square removed can be tiled by IH.

For other 3, remove corners which can be covered by L-shaped piece & tile by IH.

So to tile $2^n \times 2^n$ board, use strategy from IH & add L-block in corners.

Strong induction

We've been showing: $\forall n P(n)$

- ① $P(1)$
- ② $P(k-1) \rightarrow P(k)$

Strong induction is similar:

- ① $P(1)$
- ② $[P(1) \wedge P(2) \wedge \dots \wedge P(k-1)] \rightarrow P(k)$

We just use more information in our inductive step.

Ex: Show that any integer $n > 1$ can be written as a product of prime numbers.

proof: Induction on n

Base: $n=2$ ✓

IH: Assume any number $\leq n-1$ can be written as product of primes

IS: Consider n .

Case 1: If n is prime, done. -

Case 2: Suppose n is not prime.
Then there exist $1 < a, b \leq n-1$
such that $n = a \cdot b$.

By IH, a & b can be written as
a product of primes:
 $a = p_1 p_2 \dots p_e$, $b = q_1 \dots q_m$

So

$$n = a \cdot b = p_1 \dots p_e q_1 \dots q_m$$

So n can also be written as
a product of primes.

Why strong induction?
Here, $n-1$ doesn't help us.



Ex: Prove that every amount of postage of 12 cents or more can be formed using 4 + 5 cent stamps.

proof:

Base case: $n=12$

Use 4, 4, 4

$n=13$: 4, 4, 5

$n=14$: 4, 5, 5

$n=15$: 5, 5, 5

IH: Any postage between 16 + $n-1$ can be formed using 4 + 5 cent stamps.

IS: Make postage for n .

Make postage for $n-4$, + $n-4 \geq 12$ so by IH can make postage w/ these stamps.