Note Title 10/7/2010

Math 135 - Big-Omeage & Big-Theta

Announcements

- Next HW is posted
  due Wed after fall break (20^{th})

Big-Omega

Dfn: Let f & g be functions from $\mathbb{R} \to \mathbb{R}$ (or $\mathbb{Z} \to \mathbb{R}$)
We say $f(x)$ is $\Omega(g(x))$ if $\exists$ positive constants C, k such that

$$|f(x)| \geq C \cdot |g(x)| \quad \text{when } x > k.$$

(Read – f is big-Omega of g).

Ex: $\displaystyle\sum_{i=1}^{n} i = \Omega(n^2)$

$$\underbrace{\sum_{i=1}^{n} i}_{\parallel} \quad \frac{n(n+1)}{2} = \frac{n^2}{2} + \frac{n}{2} \geq \frac{n^2}{2}$$

method #1

so let $k=1$, $c=\frac{1}{2}$

$$\sum_{i=1}^{n} i = \underbrace{1+2+3+\cdots}_{\text{first half}} + (n-1) + n \geq \underbrace{\frac{n}{2} + \left(\frac{n}{2}+1\right) + \cdots + (n-1) + n}_{n/2 \text{ things left}}$$

$$\& \quad \frac{n}{2} + \left(\frac{n}{2}+1\right) + \cdots + n \geq \underbrace{\frac{n}{2} + \frac{n}{2} + \cdots + \frac{n}{2}}_{n/2 \text{ times}} = \frac{n}{2} \cdot \frac{n}{2} = \frac{n^2}{4}$$

so let $k=1$ & $c=\frac{1}{4}$

$$1 + 2 + \cdots + n \geq (n-3) + (n-2) + (n-1) + n$$

$$= 4n - 6 = \Omega(n)$$

Could have used $\frac{n}{3}$;

$$1 + 2 + \cdots + n \geq \underbrace{\frac{n}{3} + \left(\frac{n}{3}+1\right) + \cdots + (n-1) + n}_{\frac{2n}{3}}$$

$$\geq \underbrace{\frac{n}{3} + \frac{n}{3} + \cdots + \frac{n}{3}}_{2n/3} = \frac{n}{3} \cdot \frac{2n}{3} = \frac{2n^2}{9}$$

So let $k=1$, $c = \frac{2}{9}$

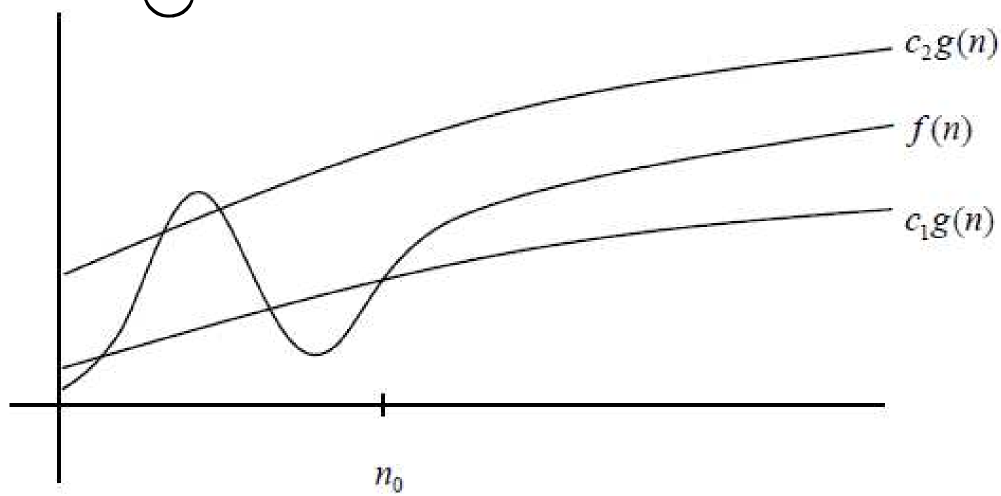
Ex: $5x^2+2x+1 = \Theta(x^2)$

Big-Theta

Let f & g be functions ($\mathbb{R} \to \mathbb{R}$ or $\mathbb{Z} \to \mathbb{R}$).
We say f(x) is $\Theta(g(x))$ if

- f(x) is $\Omega(g(x))$
- f(x) is $O(g(x))$

We say f & g are asymptotically equivalent.



Ex: $\sum_{i=1}^{n} i = \Theta(n^2)$.

Why? We just showed $\sum_{i=1}^{n} i = \Omega(n)$ $\frac{1}{4}$

Last time, we showed $\sum_{i=1}^{n} i = O(n)$

2 or 3?