Math 135: Discrete Mathematics, Fall 2010
Homework 7

Due *in class* on Nov. 29, 2010

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1. Answer the following questions - and be sure to explain your answers.

   (a) The name of a variable in the C programming language is a string that can contain uppercase letters, lowercase letters, digits, or underscores. Further, the first character much be a letter (either upper or lower case) or an underscore. If the name of a variable is determined only by its first 8 characters (the rest are ignored), how many different variables can be named in C?

   (b) How many ways are there for 10 women and 6 men to stand in a line so that no two men stand next to each other?

   (c) A bowl contains 10 red balls and 10 blue balls. A woman selects balls at random without looking at them. How many balls must she select in order to be sure of having at least 3 balls of the same color?

   (d) A professor write 40 discrete math True/False questions for a text. If exactly 17 of these problems are true and questions can go in any order, how many different answer keys are possible?

   (e) Seven women and 9 men are on the faculty of a math department. How many ways are there to select a committee of 5 members of the department if at least one woman must be on the committee? What if at least one woman and one man must be on the committee?

   (f) How many bit strings of length 10 contain either 5 consecutive 1's or 5 consecutive 0's?

   (g) What is the coefficient of x^8y^9 in the expansion of $(3x+2y)^{17}$?

2. Give a combinatorial proof of the following identity:

   $\binom{n}{r}\binom{r}{k} = \binom{n}{k}\binom{n-k}{r-k}$

3. (a) Show that if $n+1$ integers are chosen from the set $\{1, 2, \ldots, 3n\}$, then there are always two which differ by at most 2.

   (b) Show that if $n+1$ integers are chosen from the set $\{1, 2, \ldots, m \cdot n\}$, then there are always two which differ by at most $m-1$.