

Math 135: Discrete Mathematics, Fall 2010

Homework 8

Due *in class* on Monday, November 15, 2010

1. Solve the following *asymptotically*. You need to argue why your bounds are correct (either by induction or using summations and recursion trees or using Master theorem). However, you do not need to prove that your big-O bounds are formally correct; for example, if you get a bound of $n/2 \log n + 12$, you may just say that it is $O(n \log n)$.

For each, assume small constant base cases, such as $A(1) = O(1)$ or $A(2) = O(1)$.

- (a) $A(n) = 5A(n/5) + n^3$
- (b) $B(n) = 4B(n/4) + \sqrt{n}$
- (c) $C(n) = 9C(n/2) + (3n^2 + 2)$
- (d) $D(n) = 3D(n/3) + 28n$
- (e) $E(n) = 2E(\sqrt{n}) + 1$
- (f) $F(n) = F(n/2) + F(n/4) + F(n/6) + F(n/12) + n$

2. Analyze the runtime of the following algorithms. (“Analyze the runtime” means give a simple function $f(n)$ such that the runtime of the algorithm is $\Theta(f(n))$.) You should explain how you got your answers, but you do not need to prove that your answers are correct.

(a) $\text{ALGA}(n)$:

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if  $n = 0$  or  $n = 1$  then
    return 1
else
     $x \leftarrow \text{ALGA}(n - 1)$ 
     $y \leftarrow \text{ALGA}(n - 2)$ 
    return  $2x + 3y$ 

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(b) $\text{ALGB}(A[1 \dots n])$:

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if  $n \leq 1$  then
    return 0
else
     $k \leftarrow \lfloor n/2 \rfloor$ 
     $x \leftarrow \text{ALGB}(A[1 \dots k])$ 
     $y \leftarrow \text{ALGB}(A[k + 1 \dots n])$ 
     $z \leftarrow 0$ 
    for  $i = 1$  to  $k$ 
        for  $j = k + 1$  to  $n$ 
             $z \leftarrow z + A[i] * A[j]$ 
    return  $x + y + z$ 

```

(c) $\text{ALGC}(A[1 \dots n])$:

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 $i \leftarrow \lceil n/3 \rceil + 1$ 
 $j \leftarrow \lfloor 2n/3 \rfloor$ 
if  $n = 1$  then
    return  $A[1]$ 
else
     $x \leftarrow \text{ALGC}(A[1 \dots j])$ 
     $y \leftarrow \text{ALGC}(A[i \dots n])$ 
     $z \leftarrow 0$  for  $k = i$  to  $j$  do
         $z \leftarrow z + A[k]$ 
    return  $xy + z$ 

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