Math 135: Discrete Mathematics, Fall 2010
Homework 6

Due *in class* on Friday, October 6, 2010

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1. Let f_n be the n^{th} Fibonacci number, defined as $f_n = f_{n-1} + f_{n-2}$ with $f_0 = 0$ and $f_1 = 1$. (Hint: Remember, induction is your friend when doing recurrences!)

   (a) Prove that $\sum_{i=1}^{n}(f_i)^2 = f_n f_{n+1}$ whenever n is a positive integer.

   (b) Show that $f_{n+1}f_{n-1} - (f_n)^2 = (-1)^n$ when n is a positive integer.

2. Suppose that you find on your Math 135 instructor's desk a toy with 3 wooden pegs, one of which has 8 disks of different sizes neatly stacked (largest on the bottom, smallest on the top) on it. A card near the toy tells you the rules of playing:

   - Your goal is to move all of the disks from the first peg to one of the other two pegs.
   - Only one disk may be moved at a time.
   - No disk may ever be placed on a smaller disk.

   Let R_n be the minimum number of steps required to move n disks on a peg of such a toy to another peg. Impress your instructor by giving (and justifying) a recurrence for R_n and the solving it.

3. Give *exact* solutions to the following recurrences. Show your work.

   (a) $A(n) = -4A(n-1) + 5A(n-2)$, $A(0) = 2$, $A(1) = 8$.

   (b) Find the solution to the same recurrence as part (a), with $A(0) = 2$, $A(1) = 4$.

   (c) $C(n) = 2C(n-1) + n + 5$, $C(0) = 0$.

4. Given *general form* solutions to the following recurrences. (Note: this means you don't have to solve for the constants!)

   (a) $A(n) = 7A(n-1) - 16A(n-2) + 12A(n-3) + n4^n$

   (b) $B(n) = 4B(n-1) - 4B(n-2) + (n^2+1)2^n$

   (c) $C(n) = 7C(n-2) + 6C(n-3)$