

Math 135: Discrete Mathematics, Fall 2010

Homework 4

Due *in class* on Friday, Oct. 1, 2010

1. For the following functions, decide whether each one is one-to-one, onto, and bijective, and prove each of your answers.
 - (a) $f : \mathbb{R} \rightarrow \mathbb{R}$, with $f(x) = x^2 + 1$
 - (b) $f : \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z}$, with $f(m, n) = 2m - n$
 - (c) $f : \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z}$, with $f(m, n) = m^2 - n^2$

2. Give an example of a function from \mathbb{N} to \mathbb{N} which is:
 - (a) one-to-one but not onto
 - (b) onto but not one-to-one
 - (c) both onto and one-to-one (but NOT the identity function)
 - (d) neither one-to-one or onto

3. Let f be a function from the set A to the set B . Let S and T be subsets of A .
 - (a) Prove that $f(S \cup T) = f(S) \cup f(T)$
 - (b) Prove that $f(S \cap T) \subseteq f(S) \cap f(T)$
 - (c) Give an example to show that the inclusion from part (b) may be proper - in other words, give examples of sets and a function where $f(S \cap T) \subset f(S) \cap f(T)$