Math 135: Discrete Mathematics, Fall 2010
Homework 3

Due *in class* on Wednesday, Sept. 22, 2010

For this homework, please submit your own work individually. Please make sure to read the course policies on homework *before* writing up your homework.

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1. Find two sets A and B such that $A \in B$ and $A \subseteq B$.

2. Give the following sets, if $A = \{a, b, c\}$, $B = \{(1,4), 2, \{2,3\}\}$, and $C = \{a, 1, (c, 2)\}$. (Recall that $\mathcal{P}(X)$ is the power set of X.)

   (a) $\mathcal{P}(B)$

   (b) $(A \times B) - C$

   (c) $\mathcal{P}(\mathcal{P}(\emptyset)) \times A$

3. Prove or disprove the following:

   (a) $A \times (B \cup C) = (A \times B) \cap (A \times C)$

   (b) $(A - C) \cap (C - B) = \emptyset$

   (c) If $\mathcal{P}(A) = \mathcal{P}(B)$, then $A = B$.

   (d) If $A \cap C = B \cap C$, then $A = B$.

4. If A and B are sets, then the *symmetric difference* of A and B is written $A \bigtriangleup B$ and is defined to be $(A - B) \cup (B - A)$. Prove that if $A_1, A_2, \ldots, A_n$ are sets, then

   $$(((A_1 \bigtriangleup A_2) \bigtriangleup A_3) \cdots \bigtriangleup A_{n-1}) \triangle A_n = \{x \ : \ x \text{ is in an odd number of the sets } A_1, A_2, \ldots, A_n\}\,.$$

   Hint: Think induction!

5. (a) Prove that if A and B are sets, then $\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$.

   (b) Give a counterexample for the following statement: if A and B are sets, then $\mathcal{P}(A) \cup \mathcal{P}(B) = \mathcal{P}(A \cup B)$.