Math 135: Discrete Mathematics, Fall 2010
Homework 2

Due *in class* on Monday, Sept. 13, 2010

For this homework, you may write up solutions with 1 partner; both of you will receive the same grade based on your joint writeup. Please make sure to read the course policies on homework *before* writing up your homework.

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1. Prove that if n is a positive integer, then n is even if and only if $7n + 4$ is even.

2. Prove that if m and n are integers and mn is even, then m is even or n is even.

3. Prove or disprove that the product of a (nonzero) rational number and an irrational number is irrational.

4. Prove that the square of an integer ends with 0, 1, 4, 5, 6, or 9. (Hint: Let $n = 10k + l$ where $l = 0, 1, 2, \ldots, 9$).

5. (a) Prove that $3^n < n!$ if n is an integer greater than 6.
   (b) Prove that $n! < n^n$ if $n \geq 1$

6. Prove that $\sum_{i=1}^{n} i \cdot i! = (n+1)! - 1$ whenever n is a positive integer. (Recall that $\sum_{i=1}^{n} i = \cdot i! 1 \cdot 1! + 2 \cdot 2! + \cdots + n \cdot n!$).

7. Assume that a chocolate bar consists of n squares arranged in a rectangular pattern. The bar can only be broken along vertical or horizontal lines separating the squares. (Think of a Hershey's bar.)

   Assuming that only one piece can be broken at a time, determine how many breaks you must make in order to break the bar into n squares. Use induction to prove your answer is correct.