

# Math 135: Discrete Mathematics, Fall 2010

## Homework 1

Due *in class* on Friday, January 22, 2010

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Submit your solutions for this homework *in class* on Friday, September 9. Please make sure to read the course policies on homework *before* writing up your homework.

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1. Write the negation, contrapositive, converse and inverse of the following statements.
  - (a) For any integer  $x$ , if  $x$  is even, then  $x^2$  is even.
  - (b) You will be late to class if you oversleep.
  - (c) If Dr. Chambers' daughter is sick this week, then she either brings her to lecture or gets a babysitter.
2. Rewrite the following propositions as unambiguous English sentences, given the following prepositions.

- $A(x)$  means " $x$  likes West Wing".
- $B(x)$  means " $x$  likes Buffy".
- $C(x)$  means " $x$  has good taste".
- $D(x)$  means " $x$  watches TV".

For example the statements  $\forall x[D(x) \rightarrow A(x)]$  could be translated to "Everyone who watches TV likes West Wing."

- (a)  $\forall x[A(x) \vee B(x) \rightarrow D(x)]$
  - (b)  $\exists x[A(x) \wedge B(x)]$
  - (c)  $\forall x[B(x) \rightarrow C(x)]$
  - (d)  $\exists x[A(x) \vee B(x) \wedge C(x) \wedge D(x)]$
3. Express the **negations** of the following statements so that all negation symbols appear immediately preceding the predicates (and not outside any quantifiers or groups of predicates).
    - (a)  $\forall x \exists y P(x, y) \vee \forall x \exists y Q(x, y)$
    - (b)  $\exists x \exists y (Q(x, y) \rightarrow Q(y, x))$
    - (c)  $\forall y \exists x \exists z (T(x, y, z) \vee Q(x, y)) \wedge \exists x \forall y \neg P(x, y)$
  4. Classify the following formulas into logically equivalent groups. (Hint: Try using truth tables!)

- (a)  $p$
  - (b)  $p \vee \neg p$
  - (c)  $p \wedge \neg p$
  - (d)  $(p \wedge q) \rightarrow p$
  - (e)  $(p \vee q) \rightarrow p$
  - (f)  $((p \vee q) \wedge \neg q) \rightarrow (p \wedge q)$
  - (g)  $(p \vee \neg p) \rightarrow (p \wedge \neg p)$
  - (h)  $((p \vee q) \vee \neg q) \vee (r \wedge p) \wedge (p \vee \neg q)$
5. While walking across campus, you come across 3 people have an argument. The first, Alice, tells you, "Bob or Carol is lying." The second, Bob, tells you, "Carol is lying". The third, Carol, tells you, "Alice and I are both telling the truth". Who, if anyone, is telling you the truth?
6. **Extra Credit:** Next, you come across two different people on your walk. Donald says, "I am lying if Erik is." Erik says, "If I am lying, then Donald is lying." Can you tell who if anyone is telling the truth?