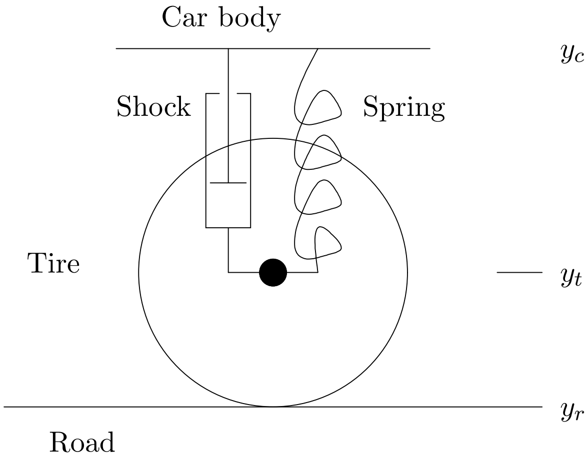
Scientific Programming

Project 2: Speed bumps and car suspensions

Due 10/1. You can work either alone in in pairs.

Project Goal To simulate how a car reacts to driving over a speed bump at different speeds.

Background Both the tires of a car and its suspension system help limit how much a car bounces when it goes over a bump. We will use a quarter-car model of the suspension: one tire and its suspension that caries a quarter of the weight of the car.



A car suspension has a spring and a shock absorber that connect the cars body to the tire's axle. The force exerted on the spring is proportional to how much it is compressed. The shock absorber damps any movement and pushes with a force proportional to the velocity of movement. If y_c is the height of the car body and y_t is the height of the tire then the force exerted by the suspension is equal to

$$k_c(y_c - y_t - l) + b_c(\frac{dy_c}{dt} - \frac{dy_t}{dt})$$

where k_c is the spring constant of the car's suspension, l is the resting length of the cars suspension and b_t is the damping constant of the car's suspension.

If you push down on a car tire you'll notice that it springs back and that the faster you push down the greater the resistance. So, a tire has features of both a spring and a damper. If a tire's center is at a height of y_t and the road is at a height y_r then the upward force on the tire is equal to

$$k_t(y_t - y_r - r) + b_t\frac{dy_t}{dt}$$

where k_t is the spring constant of the tire, b_t is the damping constant of the tire and r is its radius.

The cumulative (upward) force on the car tire is equal to force exerted by the tire minus the force of the cars suspension minus the force of gravity. Dividing this by the mass of the tire, give the vertical acceleration of the tire.

$$a_t = \frac{d^2y_t}{dt^2} = \frac{k_c(y_c - y_t - l) + b_c(\frac{dy_c}{dt} - \frac{dy_t}{dt}) - k_t(y_t - y_r - r) - b_t\frac{dy_t}{dt}}{m_t} - g$$

And the cumulative force on the car body is equal to the force exerted by the cars suspension minus the force of gravity. Dividing by the mass of the car give the (vertical) acceleration on the car body.

$$a_c = \frac{d^2y_c}{dt^2} = -\frac{k_c(y_c - y_t - l) + b_c(\frac{dy_c}{dt} - \frac{dy_t}{dt})}{m_c} - g$$

Project Assignment Suppose that the constants are defined as

Car mass supported by tire	m_c	500 kg
Suspension length	l	27 cm
Car suspension spring constant	k_c	32,000 N/m
Car suspension damping constant	b_c	7,500 N s/m
Tire mass	m_t	15 kg
Tire radius	r	34 cm
Tire spring constant	k_t	65,000 N/m
Tire damping constant	b_t	250 N s/m

The car drives over a speed bump at different speeds. The greater the acceleration and jerk (the derivative of acceleration) experienced by the passengers the more unpleasant the bump. These are measured by $\frac{d^2 y_c}{dt^2}$ and $\frac{d^3 y_c}{dt^3}$, respectively. Assume that before the car is moving the vertical velocity of the car body and tire are zero and that the vertical positions of the car body and car tire are 26.99 cm and 40.75 cm, respectively.

The class website has a Matlab file `bump.h` that contains the function `bump(x)` that indicates how high the speed bump is x meters down the road. You should use this function to calculate y_r at each stage of your calculation.

First, for a car driving 5, 10, 15, 20 and 25 miles an hour produce graphs of (all as a function of time): the height of the car tire, the height of the car body, the (vertical) acceleration experienced by the passengers and the (vertical) jerk experienced by the passengers. You should study what happens for 10 seconds. Describe any conclusions you make from examining the graphs.

Second, for cars driving 1 to 35 miles per hour graph (as a function of the speed of the car): the maximum height of the car body from its initial position, the maximum magnitude of the (vertical) acceleration of the car body and the maximum magnitude of the (vertical) jerk of the car body. Interpret the results. In particular, at what speed or speeds is going over the speed bump most violent?

Extra Credit A second file `road.m` contains the height of a continuously bumpy road. You can find the height of the road x meters from the start by calling the function `road(x)`. Determine what speed would be best to drive on this road for an extended period of time. Justify your answer for why this would be the best speed.

Requirements

- You should include you Matlab code that performs the calculations and displays the relevant graphs. It might be helpful to break this down into multiple files.
- Each of your Matlab files should have comment lines that (at a minimum) give the name of the file, your name and the purpose of the included Matlab code.
- Make sure your graphs are clearly labeled.

Hints and Suggestions

- Start early.
- Make sure to translate the cars speed to meters per second.
- For the first part you should have one set of graphs for the car going 5 mph, another set for 10 mph, etc.
- For the second part make sure that you plot the maximum magnitudes of acceleration and jerk (not their maximum values).