

# Scientific Programming

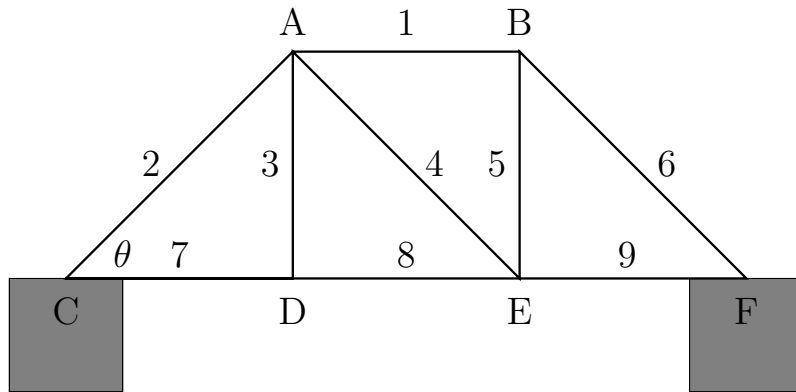
## Project 1: Capacity of a truss bridge

Due 9/12. You may work in teams of two on this project.

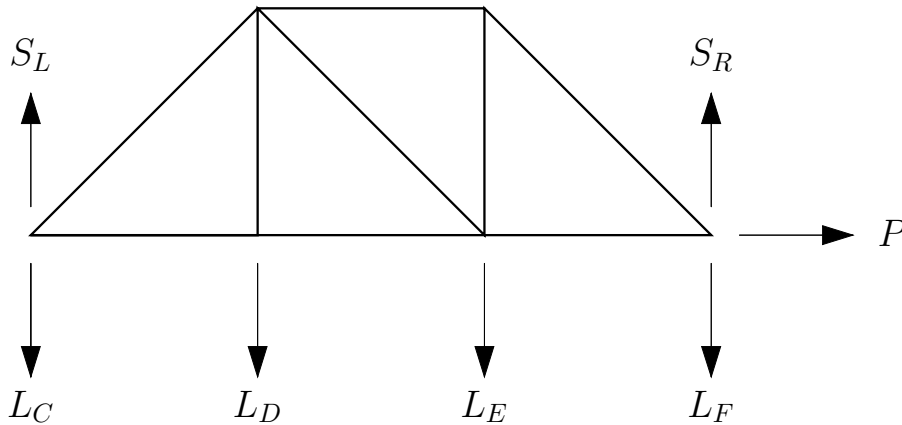
**Project Goal** To simulate the stresses put on a bridge when a heavy truck drives across it.

**Background** We will be considering a basic truss bridge made with steel beams. The combination of the weight on the bridge and any vehicles driving across puts stress on each of the elements (or beams) of the bridge. Some are under compression, others under tension. The amount of stress on the bridge can be modeled using a system of linear equations. This model is simpler than what happens in the real world, but is still used to approximate loads on bridges.

Below is a figure of a simple truss bridge. It is supported at the left and right ends. There are nine elements (or beams) of the bridge labeled 1–9 and six joints where they meet labeled A–F.



There are several external forces acting on the bridge, see below. There are loads (weight to be carried) at each of the joints,  $L_A-L_F$  that pull downward, and support loads from the left and right,  $S_L, S_R$  that hold the bridge up, and outward pressure  $P$  that pushes outward at the right support. (It is assumed that there is no pressure pushing output on the left support.)



The forces internal to the bridge are compression/tension on the bridge elements,  $f_1, \dots, f_9$ . These act along the beams with a positive sign representing tension and negative representing compression. In this model there is no torsion or twisting at the joints; however, in the real world there may be.

If the external loads on the bridge are known (but not the forces for pressure or support) it is possible to calculate all of the internal forces and remaining external forces. At each joint all of the forces need to

cancel each other out. At each joint this leads to a pair of equations, one to ensure that the horizontal forces balance and one to ensure the same for the vertical forces.

Consider joint B. There are three forces that interact at the joint  $f_1$ ,  $f_5$  and  $f_6$ . First, we will break them down into horizontal and vertical components.  $f_1$  is purely horizontal, and  $f_5$  acts purely vertically. The horizontal component of  $f_6$  is equal to  $f_6 \cos \theta$  and the vertical equal to  $f_6 \sin \theta$ , where  $\theta$  is the angle of the diagonal support shown in the first figure. This allows us to write down the equations:

$$\{\text{leftward forces}\} = \{\text{rightward forces}\} \Rightarrow f_1 = f_6 \cos \theta$$

$$\{\text{downward forces}\} = \{\text{upward forces}\} \Rightarrow f_5 + f_6 \sin \theta$$

At joint F we also need to consider external forces:  $L_F$  acts downward,  $S_R$  upward and  $P$  to the right.  $f_6$  has a horizontal component of  $f_6 \cos \theta$  and a vertical component of  $f_6 \sin \theta$ . And  $f_9$  pulls to the left. The two equations we get for this joint are:

$$\{\text{leftward forces}\} = \{\text{rightward forces}\} \Rightarrow f_9 + f_6 \cos \theta = P$$

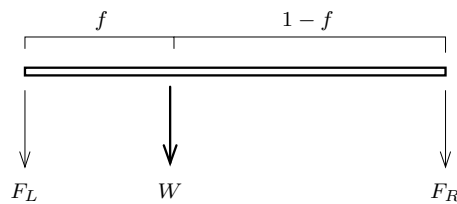
$$\{\text{downward forces}\} = \{\text{upward forces}\} \Rightarrow L_F = S_R + f_6 \sin \theta$$

We can do similar calculations at all of the joints to get a total of 12 equations.

Assuming that there are only loads on the bottom joints and that all of the triangles have the same angles, we get the following equations.

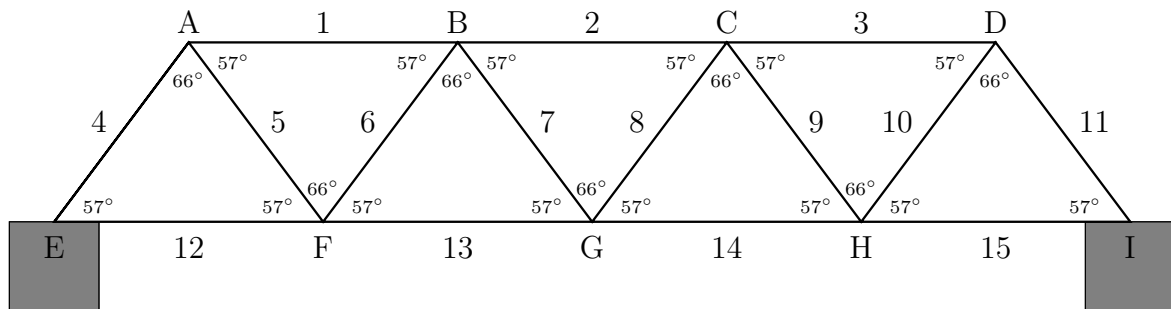
$$\begin{aligned} \text{A: } & f_2 \cos \theta = f_1 \\ & f_2 \sin \theta + f_3 + f_4 \sin \theta = 0 \\ \text{B: } & f_1 = f_6 \cos \theta \\ & f_5 + f_6 \sin \theta = 0 \\ \text{C: } & 0 = f_2 \cos \theta + f_7 \\ & L_A = S_L + f_2 \sin \theta \\ \text{D: } & f_7 = f_8 \\ & L_D = f_3 \\ \text{E: } & f_8 + f_4 \cos \theta = f_9 \\ & L_E = f_4 \sin \theta + f_5 \\ \text{F: } & f_9 + f_6 \cos \theta = P \\ & L_F = S_R + f_6 \sin \theta \end{aligned}$$

There are a total of 12 equation involving 12 unknown variables:  $f_1, \dots, f_9, S_L, S_R$  and  $P$ . Solving this system of linear equations will tell you all of the internal and external forces involved.



In the previous example, we only considered weight (or load) placed at a joint. It is also possible to have load in the middle of a beam. Any load at the center of a beam is evenly dispersed between its two ends. So a 50 kg load in the center of a beam would be the same as a 25 kg load at each end. In general, a weight,  $W$ , that is a fraction  $f$  from the left side a beam pulls down with a force of  $(1 - f)W$  on the left end of the beam and  $fW$  on the right end of the beam. For example, if a weight of 14 kg is attached 2 meters from the left end of a 5 meter beam then the weight is  $f = \frac{2}{5}$  of the way from the left end of the beam. This means that the left end of the beam is pulled down with a force of  $(1 - \frac{2}{5})14 = 8.4$  kg and the right end of the beam is pulled down with a force of 5.6 kg. Notice that the total of these two forces is equal to the weight pulling down on the beam.

**Project Assignment** Below is the truss design for a bridge. The bridge is 36 meters in length and the truss breaks this evenly into four 9 meter segments. The bridge structure puts load on each joint of the bridge. The weight of the trusses put a load of 1,500 kg on each of the four joints at the top of the structure. The combination of the weights of the trusses and road deck put a load of 5,000 kg on each of the five joints at the bottom of the structure.



A two-axle truck drives across the bridge. It weighs 12,000 kg; with the weight evenly divided between its axles. The axles are 5 m apart.

Perform your calculations with the truck in at least 5 different positions on the bridge. What is the maximum compression and maximum tension on each bridge member? At what position on the bridge does the truck put the most stress on the bridge structure? How much load is put on the two bridge supports? Make at least 4 separate graphs: the load on the top beams of the bridge, the load on the bottom beams of the bridge, the load on the diagonals and the loads on the bridge supports.

### Requirements

- You should include your Matlab code that performs the calculations and displays the relevant graphs. It might be helpful to break this down into multiple files.
- Each of your Matlab files should have comment lines that (at a minimum) give the name of the file, your name and the purpose of the included Matlab code.
- You should have approximately 1 page of explanation of how you arrived at your results and interpreting them. (This does not count the space that your Matlab code or graphs take.)
- Make sure your graphs are clearly labeled.

### Hints and Suggestions

- Start early.
- Make sure that you understand the sample project on resistor networks; it uses many of the same Matlab techniques. (Note: this project is more complicated.)
- Start early.
- You can export Matlab plots as graphics files which can be imported into a word processor when you write your report.
- Start early.
- When the truck is not on the bridge all of the loads are completely symmetric (between left and right), so your answer should also be. This means that the supporting force on the two ends should be the same, members 1 and 3 should have the same force, members 4 and 11 should have the same force, etc. Note: this is only true when the truck is not on the bridge.
- Did we mention that you should start early?