

Scientific Programming

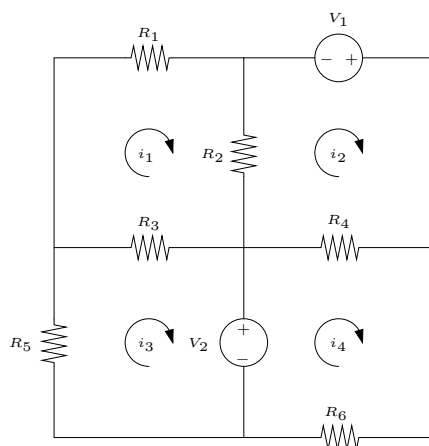
Sample Project: Kirchhoff's Law

This is a small sample project that should give you an idea of what a project writeup should look like. Keep in mind, this is going to be smaller than typical projects, so we will generally expect a longer explanation and additional content.

1 Project Assignment

Project Goal To examine the behavior of an electrical circuit as the resistances change. This uses Kirchhoff's voltage law.

Background Below is an electrical circuit consisting of resistors and voltage sources (e.g. batteries). Each voltage source provides a source of electricity measured in volts(V). Resistors inhibit the flow of electricity. The resistance is measured in Ohms(Ω).



The currents around each loop in the circuit are labeled i_1, i_2, i_3, i_4 and go clockwise. Current is measured in Amps (A). Kirchhoff's law states that the voltage around a closed circuit is zero. The voltage decrease across a resistor is equal to the current flowing across it multiplied by its resistance. For example, at resistor R_1 there is a current of i_1 across the resistor so the voltage drop is equal to $R_1 i_1$. At resistor R_2 the current is across the resistor is $i_1 - i_2$ (since the currents flow in opposite directions), so the voltage drop across the resistor is equal to $R_2(i_1 - i_2)$. For each loop we get a linear equation equal to sum of the voltages on the batteries minus any voltage drops at the resistors. This yields four equations:

$$\begin{aligned} -R_1 i_1 - R_2(i_1 - i_2) - R_3(i_1 - i_3) &= 0 \\ V_1 - R_4(i_2 - i_4) - R_2(i_2 - i_1) &= 0 \\ -V_2 - R_5(i_3) - R_3(i_3 - i_1) &= 0 \\ V_2 - R_4(i_4 - i_2) - R_6 i_4 &= 0 \end{aligned}$$

Project Assignment Using the circuit above, assume that we know all of the voltages for the batteries and the resistance for all but R_6 .

$$V_1 = 10V, V_2 = 15V, R_1 = 6\Omega, R_2 = 10\Omega, R_3 = 5\Omega, R_4 = 4\Omega, R_5 = 8\Omega$$

Assume that R_6 resistance varies from 0Ω to 20Ω . Graph the current across resistor R_1 with respect to the changing resistance of R_6 .

Hints and Suggestions You will have to do a little of rewriting of the equations so put them in terms of the unknowns (the currents). Use the `linsolve` command to solve each set of equations.

2 Project Report

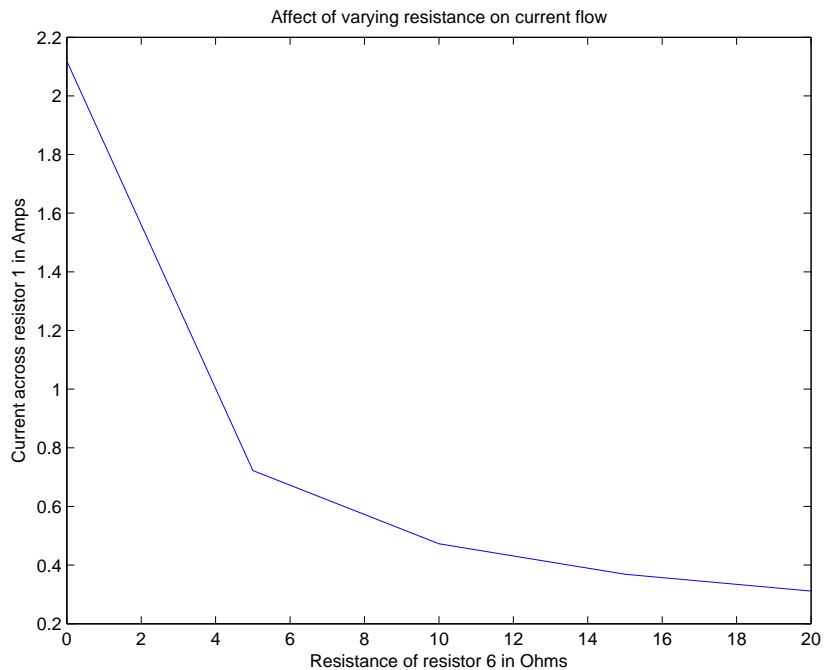
Setting up the equations We can rewrite the above equations with i_1, \dots, i_4 as the unknowns as below:

$$\begin{array}{rcccccc} (-R_1 - R_2 - R_3)i_1 & + & R_2i_2 & + & R_3i_3 & & = 0 \\ -R_2i_1 & + & (-R_2 - R_4)i_2 & & & + & R_4i_4 & = -V_1 \\ R_3i_1 & & & & (-R_3 - R_5)i_3 & & & = V_2 \\ & & R_4i_2 & & & + & (-R_4 - R_6)i_4 & = -V_2 \end{array}$$

Simulating current flow Two Matlab scripts were used to solve this problem. The first `calc_currents.m` sets up the matrix and arrays needed to solve the system of linear equations above. This script assumed the values for all of the voltages and resistances were already input and solves the system using `linsolve`.

The main script file, `Kirchhoff.m`, adjusts the value of resistance of resistor six and uses `calc_currents` to find the currents. The resulting currents across resistor one are stored in an array to be plotted.

Results Below is the plot of how the current across resistor 1 changes when the resistance of resistor varies from 0 to 20 Ohms. Notice that the current decreases as the resistance increases.



Matlab Code The Matlab script for solving, called `calc_currents.m` is bellow

```
% File calc_currents.m
% This solves for the current in the sample project using Kirchhoff's law
A = [ (-R1-R2-R3) R2 R3 0 ;
      R2 (-R2-R4) 0 R4 ;
      R3 0 (-R3-R5) 0 ;
      0 R4 0 (-R4-R6) ];
b = [ 0; -V1; V2; -V2 ];
i = linsolve(A,b);
```

The main script for the project is below:

```
% File Kirchoff.m
% Uses Kirkhoff's law to calculate the current across a resistor network for varying
% resistances and plots the results. Note this needs the file calc_current.m to
% work properly

% Setup the common values for voltages and currents
V1=10; V2=15;
R1=6; R2=10; R3=5; R4=4; R5=8;

% Setup arrays to store the results of the calculations in (these will be graphed later)
resistances = [0:5:20];
currents = zeros(5, 1);

% Deal with R6=0
R6=0;
calc_currents
currents(1) = i(1);

% Deal with R6=5
R6=5;
calc_currents
currents(2) = i(1);

% Deal with R6=10
R6=10;
calc_currents
currents(3) = i(1);

% Deal with R6=15
R6=15;
calc_currents
currents(4) = i(1);

% Deal with R6=20
R6=20;
calc_currents
currents(5) = i(1);

% Plot the results
plot(resistances, currents)

% Add labels, titles, etc. to graph
xlabel('Resistance of resistor 6 in Ohms')
ylabel('Current across resistor 1 in Amps')
title('Affect of varying resistance on current flow')
```